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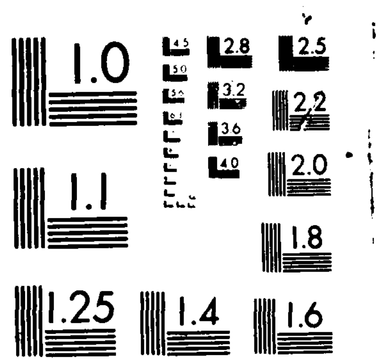

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The propagation of Raman solitons in media with homogeneous and inhomogeneous broadening is studied. Results are obtained for the amplitude and temporal width of solitons as functions of propagation distance and detuning from exact Raman resonance. The analytical treatment uses differential conservation laws, which give good quantitative estimates as well as a detailed understanding of the balance of physical processes responsible for soliton evolution. Numerical calculations are used to verify the analytical results and to extend them to the transient regime, where the soliton width is comparable to the coherence time of the medium. Solitons are found to be more stable for inhomogeneous broadening than for homogeneous broadening. In order to obtain both good pulse narrowing and soliton stability, the detuning from exact Raman resonance should not exceed 5% of the Raman line width. These results make media with broad inhomogeneous Raman lines like optical fibers the most promising candidates for the application of Raman solitons.

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SOLITONS IN STIMULATED RAMAN SCATTERING:

GENERATION AND CONTROL OF ULTRASHORT LIGHT PULSES

FINAL REPORT

KAI J. DRUHL

JUNE 15, 1987

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3. STATEMENT OF PROBLEM

Solitons in stimulated Raman scattering are a coherent transient phenomenon, in which short pulses of coherent (pump) radiation are created in an envelope of longer wave length (Stokes) radiation.

In a Raman active medium with homogeneous broadening solitons can be created from a large class of initial conditions, including those appropriate for spontaneous Raman scattering and for Raman amplifiers with weak Stokes probe. The resulting pulse of pump radiation will initially have a width comparable to the coherence decay time.

The problem addressed in the work reported here is to determine the detailed dynamics of soliton propagation in media with homogeneous and inhomogeneous broadening. Of particular interest are the rates of soliton narrowing and decay and the frequency and temporal characteristics of the developing optical pulse. Soliton decay occurs in broadened media, if the two optical beams are not exactly in resonance with the Raman transition, a situation which to some extent frequently occurs in practice. It is hence important to determine the limitations on soliton stability imposed by this effect.

Both numerical and analytical methods have been employed.

4. SUMMARY OF RESULTS

A. RESULTS FOR NARROW SOLITONS (HYPER TRANSIENT REGIME)

If the temporal soliton width is much smaller than the coherence decay time, broadening can be treated as a perturbation. Analytical results have been obtained for this case by using the method of asymptotic perturbation theory, based on the inverse scattering transform (IST) and also a method based on constants of motion, which is more directly related to the physics of the problem. Both approaches give the same results.

The following results are obtained for homogeneously broadend media:

For exact Raman resonance the width decreases with gain according to an inverse square root law, and becomes independent of the initial width in the large gain limit. For off resonant Raman scattering the soliton amplitude, defined as the maximum relative pump intensity, decreases exponentially with increasing gain. Since a relation exists between amplitude and frequency detuning, this means that the average detuning increases in the process of propagation. The decay coefficient is proportional to the square of the initial frequency mismatch. The width decreases more slowly than in the resonant case, reaches a minimum at an amplitude of 0.5 and increases beyond that point.

Numerical studies confirm these results. The agreement is excellent for exact resonance, and very good for the off resonant case. In the latter case at large gains the rate of soliton decay is found to be somewhat smaller than predicted, while the rate of soliton narrowing is somewhat larger. The reason appears to be an almost linear frequency chirp, which develops between the leading edge (lower frequency) and trailing edge (higher frequency) of the soliton. This is a second order effect which cannot be modelled in the approximation employed. The effect is small however, as long as the detuning is small compared to the inverse soliton width, even if it should be larger than the Raman line width.

For inhomogeneously broadend media analytical results have been obtained for the resonant case, and numerical results for both resonant and off resonant case:

In the resonant case the soliton width decreases very weakly with gain like the inverse of the third root. This is due to the fact that for inhomogeneous broadening the frequency spectrum is more sharply localized. Hence the temporal autocorrelation function has vanishing slope at the origin, and narrowing occurs only to second order in the width. The predictions agree excellently with numerical results.

In the off resonant case two different cases occur. If the frequency detuning is small compared to the line width, weak soliton decay and narrowing occurs. If the detuning exceeds the line width, however, complicated envelope modulations of the optical pulses occur. Soliton amplitude and width go through cycles of decrease and increase, with an overall tendency for decay and pulse broadening. An analytical theory for this type of behavior has not yet been developed.

B. RESULTS FOR BROAD SOLITONS (TRANSIENT REGIME)

In the transient regime where the soliton width is comparable to the coherence time analytical methods based on perturbation theory are no longer adequate. Numerical studies have been performed, which show that the main features found in the hypertransient case also occur in this regime. In particular pulse narrowing and decay occurs both in the homogeneously and inhomogeneously broadened case.

The rate of decay is found to be much less initially than expected from the first order theory. As a result a pulse narrowing to a width of about 20% of the coherence time is possible even in the off resonant case. A maximal frequency offset of about 5% of the Raman line width can be tolerated in this case. These results are encouraging for soliton experiments in media with broad inhomogeneous lines like methane or optical fibers.

A detailed discussion of the results under sections A. and B. above are given in the Appendices A. and B., which are self contained. Appendix A. contains a mathematical analysis of perturbation theory for the homogeneously broadened case based on soliton theory (IST). Appendix B. gives a more physical discussion of homogeneous and inhomogeneous broadening, which emphasizes the relavant physical processes, and employs differential conservation laws for photon energy and momentum.

C. SATURATING SOLITONS FOR HOMOGENEOUS BROADENING (HYPERTRANSIENT REGIME)

For sufficiently high field intensities medium saturation occurs. Raman solitons exist also in this case, and the total soliton width can be much larger than the pulse rise time, giving rise to a top hat type temporal profile. Soliton theory is quite complicated for this system, and we have studied the resonant case in the absence of Stark shift effects. Detailed results are discussed in Appendix C.

If medium saturation is strong, the rate of soliton narrowing per unit propagation distance is independent of the soliton width, and the latter decreases linearly with distance. This is much stronger than the inverse square root law found for the unsaturated case, and makes this type of Raman soliton very attractive for pulse narrowing techniques. Effects of frequency detuning and Stark shift remain to be studied.

5. LIST OF PUBLICATIONS

Kai Druhl and Gabriella Alsing, "Effect of coherence relaxation on the propagation of optical solitons: An analytical and numerical case study on asymptotic perturbation theory", Physica 20D, 429-434 (1986)

M. Yousaf, S. Shakir and K.J. Druhl, "Propagation of non resonant temporal Raman solitons in the presence of coherence decay: a perturbation theoretical analysis" (to be published, preprint is Appendix A)

K.J. Druhl, "Raman solitons in homogeneously and inhomogeneously broadened media", (to be published, preprint is Appendix B)

C. Tourenne and K.J. Druhl, "Effects of medium saturation and homogeneous broadening on the propagation of Raman solitons", (to be published, preprint is Appendix C)

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7. BIBLIOGRAPHY

M.J. Ablowitz, D.J. Kaup, and A.C. Newell, "Coherent pulse propagation: A dispersive, irreversible phenomenon", J. Math. Phys. 15, 1852-1858 (1974);

J.R. Ackerhalt and P.W. Milonni, "Solitons and four-wave mixing", Phys. Rev. A 33, 3185-3198 (1986);

F.Y.F. Chu and A.C. Scott, "Inverse scattering transform for wave-wave scattering", Phys. Rev. A 12, 2060-2064 (1975);

K.J. Druhl, R.G. Wenzel, and J.L. Carlsten, "Observation of solitons in stimulated Raman scattering", Phys. Rev. Lett. 51, 1171-1176 (1983);

K.J. Druhl and G. Alsing, "Effect of coherence relaxation on the propagation of optical solitons: An analytical and numerical case study on asymptotic perturbation theory", Physica 20D, 429-434 (1986);

K.J. Druhl, J.L. Carlsten, and R.G. Wenzel, "Aspects of soliton propagation in stimulated Raman scattering", J. Stat. Phys. 39, 615-620 (1985);

J.N. Elgin and T.B. O'Hare, "Saturation effects in transient stimulated Raman scattering", J. Phys. B: Atom. Molec. Phys. 12, 159-168 (1979);

S. Elyutin and J.N. Elgin, "Comment on recent observation of solitons in stimulated Raman scattering", Opt. Commun. 60, 104-106 (1986);

J.C. Englund and C.M. Bowden, "Spontaneous generation of Raman solitons from quantum noise", Phys. Rev. Lett. 57, 2661-2663 (1986);

G.I. Kachen and W.H. Lowdermilk, "Self-induced gain and loss modulation in coherent, transient Raman pulse propagation", Phys. Rev. A14, 1472-1474 (1976);

V.I. Karpman, "Soliton evolution in the presence of perturbation", Physica Scripta 20, 462-478 (1979);

D.J. Kaup, "Creation of a soliton out of dissipation", Physica 19D, 125-134 (1986);

D.J. Kaup and A.C. Newell, "Solitons as particles, oscillators, and in slowly changing media: A singular perturbation theory", Proc. R. Soc. Lond. A 361, 413-446 (1978);

D.J. Kaup, "A perturbation expansion for the Zakharov-Shabat inverse scattering transform", SIAM J. Appl. Math. 31, 121-133 (1976);

D.J. Kaup, "The method of solution for stimulated Raman scattering and two-photon propagation", Physica 6D, 143-154 (1983);

M.J. Konopnicki and J.H. Eberly, "Simultaneous propagation of short different-wavelength optical pulses", Phys. Rev. A 24, 2567-2583 (1981);

G.L. Lamb, Jr., "Elements of soliton Theory", John Wiley and Sons, Inc., New York, 1980;

S.L. McCall and E.L. Hahn, "Self-induced transparency", Phys. Rev. 183, 457-485 (1969);

C.J. McKinstrie and D.F. DuBois, "Relativistic solitary-wave solutions of the beat-wave equations", Phys. Rev. Lett. 57, 2022-2025 (1986);

M.G. Raymer, I.A. Walmsley, J. Mostowski and B. Sobolewska, "Quantum theory of Stokes pulse energy fluctuations", J. Opt. Soc. Am. B 1, 547 (1984);

H. Steudel, "Solitons in stimulated Raman scattering and resonant two-photon propagation", Physica 6D, 155-178 (1983);

H. Steudel, "Stimulated Raman scattering with an initial phase shift: The pre-stage of a soliton", Opt. Commun. 57, 285-289 (1986);

N. Tan-no, T. Shirahata, K. Yokoto, and H. Inaba, "Coherent transient effect in Raman pulse propagation", Phys. Rev. A 12 159-168 (1975);

N. Tan-no and Y. Higuchi, "Solitary wave solutions in coherent two-photon pulse propagation", Phys. Rev. A 16, 2181-218[illegible] (1977)

R.G. Wenzel, J.L. Carlsten, and K.J. Druhl, "Soliton experiments in stimulated Raman scattering", J. Stat. Phys. 39, 621-632 (1985);

APPENDIX A

PROPAGATION OF NON RESONANT TEMPORAL RAMAN SOLITONS IN THE PRESENCE OF COHERENCE DECAY: A PERTURBATION THEORETICAL ANALYSIS

by

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ABSTRACT

The propagation of a soliton in a Raman medium in the presence of collisional decay of coherence and detuning from Raman line center is studied. The spatial dependence of soliton width, position and amplitude is calculated by treating the decay of coherence as a perturbation. Two approaches are used, one based on the inverse scattering transform and the other employing constants of motion. Both give identical results which are confirmed through the numerical integration of the equations of Raman scattering. These results confirm the phenomena of soliton narrowing and decay, which have been observed experimentally.

INTRODUCTION

The occurrence of solitons in a Raman medium has been demonstrated experimentally [1]. We present here the results of an analytical study of the evolution of these solitons when they propagate in the Raman medium in the presence of decay of coherence and detuning from the Raman line center. Decay of coherence (which arises due to collision of molecules in a gaseous medium) is included as a perturbation to the exactly solvable equations of transient stimulated Raman scattering in which this effect is ignored. The perturbation appears in the equation for the time evolution of the medium variable. On the other hand the physical problem is to find the spatial evolution of the optical fields (which are given as functions of time at some initial position. For this reason earlier formulations of the problem [2,3] cannot be directly applied, and a reformulation of the Zhakarov Shabat problem in terms of the optical fields is necessary.

Such a reformulation has been given by Kaup [4], and applied to the case of purely imaginary eigen values (which corresponds to the physical situation of exact Raman resonance. Earlier results on soliton narrowing [5,6] were confirmed in this way. However the case of general complex eigenvalues (corresponding to off resonant fields has not been treated so far.

In this paper we analyze this general case by using asymptotic perturbation theory and working directly with the modified equation for the spatial evolution of the soliton potential, which is a nonlocal function of the optical fields. Equations for soliton width and amplitude are also derived from the first two conservation laws for the solvable case. Both approaches give identical results, which agree very well with numerical results from a direct integration of the original non linear equations.

THEORY

The process of transient stimulated Raman scattering is described by the following equations for the slowly varying amplitudes for the pump and the Stokes fields (A_P and A_S), and the off diagonal matrix element for the polarization of the medium, X [1].

$$A_{P\chi} = X A_S \tag{1}$$

$$A_{S\chi} = X^* A_P \tag{2}$$

$$X_\tau = \epsilon X - A_P A_S^* \tag{3}$$

The coordinates χ and τ are the propagation distance travelled in the gain medium and the temporal position of the soliton in the optical pulse. Here and in the following we denote partial differentiation with respect to these variables by a subscript. The first term in equation (3) describes the decay of coherence of the polarization of the medium with a rate ϵ. This term is assumed small compared to the rate of change of polarization of the medium and is treated as a perturbation to the exactly solvable equations. Equations (1) to (3) for the case $\epsilon = 0$ have been solved by the IST [2,4]. The solutions to equations (1) to (3) which we shall study here are the travelling waves in one soliton form. They are given by

$$X = \mu_I \exp(iB)\,\mathrm{sech}(A), \tag{4}$$

$$A_S = -\sqrt{\mu_I \omega_R}\,(\omega_R \tanh(A) - i\,\omega_I), \tag{5}$$

$$A_P = \sqrt{\mu_I \omega_R}\,\exp(iB)\,\mathrm{sech}(A), \tag{6}$$

where

$$A(\tau, \chi) = \omega_R \tau - \alpha, \qquad B(\tau, \chi) = \omega_I \tau + \delta. \tag{7}$$

The constants ω_R and ω_I determine the temporal width and detuning of the fields from exact Raman resonance. They are related to the real and imaginary part of the eigenvalue ζ which characterizes the one soliton solution in the framework of IST [2,4].

$$\omega_R + i\,\omega_I = 2i\,\zeta_1 = 2\,\eta_1 + 2i\,\xi_1. \tag{8}$$

The parameters α and δ are related to the residue at the corresponding pole of the transmission coefficient. They determine the temporal position and the phase of the soliton. When $\epsilon = 0$, the parameters ω_R and ω_I are constants and α and δ are linear in χ:

$$\alpha = \mu_I \chi, \qquad \delta = \mu_R \chi, \tag{9}$$

$$\mu_I = \omega_R / (\omega_R^2 + \omega_I^2), \qquad \mu_R = \omega_I / (\omega_R^2 + \omega_I^2).$$

In the presence of the perturbation ($\epsilon \neq 0$), however, they are assumed to be general functions of χ. We shall calculate these both by asymptotic perturbation theory [3] and from constants of the motion [5]. As a first step we obtain the equation for the spatial dependence of the soliton potential q. The latter is defined in terms of the optical field amplitudes as [4]

$$q = 2i\,[\exp(i\gamma)(\sin\beta)_\tau / \cos\beta]$$

$$= 2i\,\exp(i\gamma)(i\,\theta_\tau \sin\beta + \beta_\tau), \tag{10}$$

where the angular variables are defined in terms of the optical fields as:

$$A_P A_P^{*} - A_S A_S^{*} = \cos\beta \quad , \quad 2 A_P A_S^{*} = \exp(i\theta)\sin\beta ,$$

$$\gamma_\tau = \theta_\tau \cos\beta . \tag{11}$$

The spatial dependence of q is given by a zero order term corresponding to the solvable case (ϵ =0) and an additional term for ϵ >0:

$$q_\chi = q_\chi^{(0)} + q_\chi^{(1)} . \tag{12}$$

For the additional term we find:

$$q_\chi^{(1)} = i\, \gamma_\chi^{(1)} q - 2\, i \exp(i\gamma)(i\theta_\chi \sin\beta + \beta_\chi) , \tag{13}$$

$$\gamma_{\chi\tau}^{(1)} = 2\, \theta_\chi \sin^2\beta \cos\beta .$$

To first order in ϵ we can use the one soliton solution (5) and (6) together with relations (9) to calculate the corresponding angular variables. For comparison with earlier work it is convenient to rewrite the result in the form:

$$q^{(1)} = - \vartheta\, q , \tag{14}$$

where

$$\vartheta = \epsilon\, [\mu_P^2 - \mu_S^2 + 2\, i\, \mu_P \mu_S (2 + 3 \tanh(A))] \tag{15}$$

Note that ϑ here depends explicitly on both spatial and temporal variables. In this case we obtain the following result from Kaup's general equations for the eigenvalue ζ_1 and the parameters α and δ

$$i\, \zeta_{1\chi} = \eta_1^2 \int d\tau\, [\dot{\vartheta} \exp(-A) + \vartheta \exp(A)]\, \mathrm{sech}(A) , \tag{16}$$

$$\eta_{1\chi}\ \ \eta_1 [\text{illegible}]\, i\,\delta_\chi + \alpha_\chi = \tag{17}$$

$$- \eta_1 \int d\tau\, [\, \vartheta \exp(A) + \dot{\vartheta} \exp(-A)\ [\text{illegible}]\ 1 - 2A$$

$$\exp(A)\, \vartheta - \dot{\vartheta}\ [\text{illegible}]\ \mathrm{sech}(A) \,] .$$

Substituting equation (15) into these expressions we obtain the following differential equations for the soliton parameters

$$(d/d\chi)\ \omega_P = - 2\epsilon\, \omega_P\, (\mu_P^2 - \mu_S^2) \tag{18}$$

$$(d/d\chi)\ \omega_S = 4\epsilon\ \omega_P\, \mu_P \mu_S \tag{19}$$

$$(d/d\chi)\ \alpha = - 2\epsilon\, (\mu_P^2 - \mu_S^2)\,\alpha + [\text{illegible}]\ (d/d\chi)\ [\text{illegible}]\ \chi \tag{20}$$

$$(d/d\chi)\,\delta = 4\epsilon\,\mu_R\,\mu_I(1-\alpha) + (d/d\chi)\,\delta^{(0)} . \qquad (21)$$

Equations (18) and (19) can also be obtained from the first two constants of motion for this problem. The constants and their particular values for the solution (4) to (6) are given by

$$c_1 = -(1/2)\int d\tau\; q\, q^{*} = -\,\omega_R\,) \qquad (22)$$

$$c_2 = -(1/4)\int d\tau\; q\, q^{*}_{\tau} = (1/2)\,\omega_R\,\omega_I . \qquad (23)$$

The spatial derivatives of both coefficients in the presence of the perturbation can be calculated from equation (14) and give equations equivalent to (18) and (19) above. This approach can be extended to give an equation for the temporal soliton position (parameter α above), which is equivalent to equation (20). The temporal dependence of the soliton phase (parameter δ above) however cannot be determined in this way.

The solutions of equations (18) and (19) are most easily expressed in terms of the parameter μ_R and the relative intensity ρ of the pump field at soliton center

$$\rho = \mu_I\,\omega_R = \omega_R^2/(\omega_R^2 + \omega_I^2)$$

$$(d/d\chi)\,\mu_R = 0 , \qquad (d/d\chi)\,\rho = -4\epsilon\,\mu_R^2\,\rho . \qquad (24)$$

As mentioned above, the coefficients ω_R and ω_I are in physical terms the inverse of the temporal width of the soliton and the relative frequency difference between the optical field amplitudes ("detuning" from exact Raman resonance). They are obtained from equations (24) as

$$\omega_R = \rho/\mu_R \qquad (25)$$

$$\omega_I^2 = \rho(1-\rho)/\mu_R^2 \qquad \text{for } \omega_I \neq 0)$$

$$\omega_R^2 = \omega_R^2(0) + 4\epsilon\chi \qquad \text{for } \omega_I = 0$$

The relative pump intensity is seen to decay exponentially with propagation distance for non zero detuning from resonance. For small detuning the decay coefficient is proportional to the square of the detuning. This explains the observed tendency of Raman solitons to decay in the presence of broadening and detuning or additional phase modulations. The behavior of the soliton width is different for zero and non zero detuning. While the width decreases steadily for zero detuning, it reaches a minimum for non zero detuning at the point where the relative pump intensity is equal to 0.5.

The temporal soliton position τ_c can be defined in terms of the parameter α. From equation (20) we obtain:

$$\omega_R \tau - \alpha = \omega_R(\tau - \tau_C), \tag{26}$$

$$\exp(4\epsilon \tau_C) = [\rho(0)/(1-\rho(0))](1-\rho)/\rho \quad \text{for } \omega_I \neq 0,$$

$$= 1 + 4\epsilon\, \mu_I(0)^2 \chi \quad \text{for } \omega_I = 0.$$

NUMERICAL RESULTS

In figures 1 to 3 we compare the results of a numerical solution of equations (1) to (3) to the perturbation analysis above. The exact one soliton form of the optical fields was used as initial condition at $\chi = 0$, while the medium polarization X was set equal to zero at $\tau = 0$. The temporal position of the fields was chosen such as to render the corresponding error smaller than one part in thousand. The parameters ρ and ω_R were determined for the numerical solution for the optical fields from the maximal relative pump intensity and the area under the temporal pump intensity curve. The constant μ_R was determined for off resonant solitons both from the pump intensity and width by equation (23) and from the temporal derivative of the relative phase between pump and Stokes amplitude (see equations (5) and (6)).

Three different cases are shown. The parameters chosen were $\omega_R = 1$ for all cases, $\epsilon = 0.1$ and $\omega_I = 0.0$ in case 1, $\epsilon = 0.1$ and $\omega_I = 0.5$ in case 2, and $\epsilon = 0.4$ and $\omega_I = 0.5$ in case 3. Cases 1 and 2 satisfy the condition that the damping term be small compared to the driving term:

$$\epsilon \mu_P \ll 1 \tag{27}$$

The maximal propagation distance was $\chi = 20$. Figures 1a and 1b show the analytical results for the square of ω_R and for the exponential function, equation (24), of the soliton position, which are straight lines. The dots indicate the numerical values, which fall almost exactly onto the theoretical lines.

Figures 2a and 2b show the same parameters for case 2 with detuning $\omega_I = 0.5$. The agreement is excellent for the soliton position and very good for the soliton width. Larger discrepancies occur in soliton width for $\chi > 10$. In this region the rate of increase in soliton width is smaller than predicted. The reason for this is mainly the behavior of the soliton phase. According to equations (5) and (6) the relative phase of pump and Stokes field shows linear time dependence in the asymptotic temporal region away from the soliton center corresponding to a constant frequency. The relative phase for the numerical solution on the other hand shows an almost linear frequency chirp with local frequency at soliton center close to the analytical result. The chirp increases with increasing propagation distance. In figures 2c and 2d we show the soliton amplitude (maximal relative pump intensity ϱ) and the constant μ_R . The latter is determined both from soliton width and amplitude (solid line) by equation (25) and from the relative phase (dotted line) by equations (5) and (6). Both methods begin to disagree at larger propagation distances.

Figures 3a to 3d show the same results for case 3, in which condition (27) for the validity of perturbation theory is no longer satisfied. The numerical results do however show the same qualitative features as in case 2. The quantitative amount of soliton decay and broadening is smaller than predicted from perturbation theory. Figure 3d shows that the parameter as determined from the soliton width disagrees strongly with the value obtained from the phase, the latter showing a much stronger local detuning (smaller μ_R) than predicted.

SUMMARY

We have obtained results for the propagation of Raman solitons in a homogeneously broadened medium by asymptotic perturbation theory and from constants of the motion. Both approaches give identical results, which agree with results obtained in earlier work for the resonant case (purely imaginary eigen values) and extend to the non resonant case (general complex eigenvalues). The experimentally observed phenomena of soliton narrowing and decay [7] are explained by these results. Numerical studies show very good agreement in the region of validity of this approach, and show that the qualitative features are predicted correctly even in the case where broadening cannot be treated as a perturbation.

One of us (K.D.) wishes to acknowledge stimulating discussions with D. Kaup on the subject presented here.

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REFERENCES

1. K.J. Druhl, R.G. Wenzel, and J.L. Carlsten: "Observation of solitons in stimulated Raman scattering", Phys. Rev. Lett. 51, 1171-1176 (1983);

2. F.Y.F. Chu and A.C. Scott: "Inverse scattering transform for wave-wave scattering", Phys. Rev. A 12, 2060-2064 (1975);

3. D.J. Kaup: "A perturbation expansion for the Zakharov-Shabat inverse scattering transform", SIAM J. Appl. Math. 31, 121-133 (1976);

4. D.J. Kaup: "Creation of a soliton out of dissipation", Physica 19D, 125-134 (1986);

5. K.J. Druhl, J.L. Carlsten, and R.G. Wenzel: "Aspects of soliton propagation in stimulated Raman scattering", J. Stat. Phys. 39, 615-620 (1985);

6. K.J. Druhl and G. Alsing: "Effect of coherence relaxation on the propagation of optical solitons: An analytical and numerical case study on asymptotic perturbation theory", Physica 20D, 429-434 (1986);

7. R.G. Wenzel, J.L. Carlsten, and K.J. Druhl: "Soliton experiments in stimulated Raman scattering", J. Stat. Phys. 39, 621-632 (1985);

FIGURE CAPTIONS

Figure 1a: The square of the inverse soliton width is shown as a function of distance for zero detuning. The initial soliton width is equal to 1 and the Raman line width is equal to 0.1. The straight solid line is the analytical result, and circles are the numerical results.

Figure 1b: An exponential function of the temporal soliton center is shown as a function of distance. Parameters are as in Figure 1a. The straight solid line is the analytical result, and circles are the numerical results.

Figure 2a: The square of the inverse soliton width is shown as a function of distance for a detuning of 0.5. The initial soliton width is equal to 1 and the Raman line width is equal to 0.1. The solid line is the analytical result, and circles are the numerical results.

Figure 2b: An exponential function of the temporal soliton center is shown as a function of distance. Parameters are as in Figure 2a. The solid line is the analytical result, and circles are the numerical results.

Figure 2c: The maximal relative pump intensity is shown as a function of distance. Parameters are as in Figure 2a. The solid line is the analytical result, and circles are the numerical results.

Figure 2d: Numerical results for the constant as a function of distance are shown. Parameters are as in Figure 2a. The solid line is the result obtained from the pump intensity profile, and circles indicate the result obtained from the local frequency profile.

Figure 3a: The square of the inverse soliton width is shown as a function of distance for a detuning of 0.5. The initial soliton width is equal to 1 and the Raman line width is equal to 0.4. The solid line is the analytical result, and circles are the numerical results.

Figure 3b: An exponential function of the temporal soliton center is shown as a function of distance. Parameters are as in Figure 3a. The solid line is the analytical result, and circles are the numerical results.

Figure 3c: The maximal relative pump intensity is shown as a function of distance. Parameters are as in Figure 3a. The solid line is the analytical result, and circles are the numerical results.

Figure 3d: Numerical results for the constant as a function of distance are shown. Parameters are as in Figure 3a. The solid line is the result obtained from the pump intensity profile, and circles indicate the result obtained from the local frequency profile.

SOLITON WIDTH

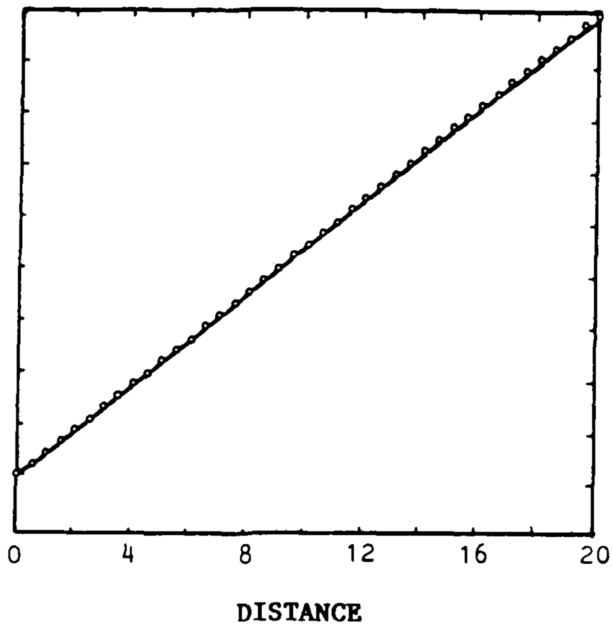


Figure 1a

SOLITON POSITION

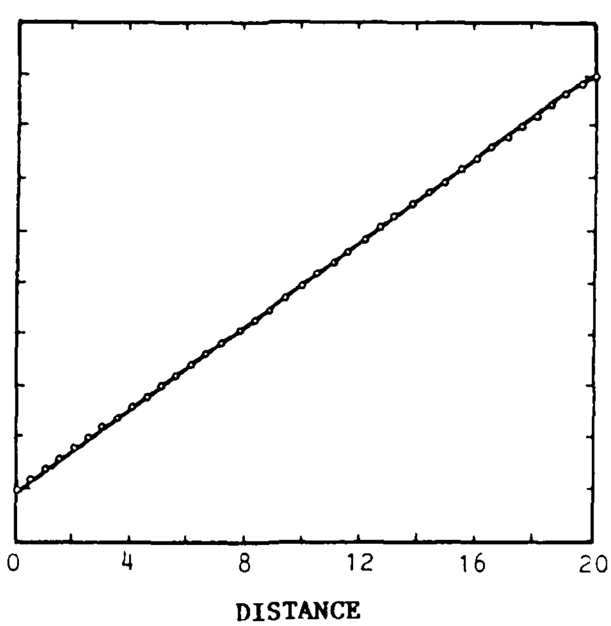
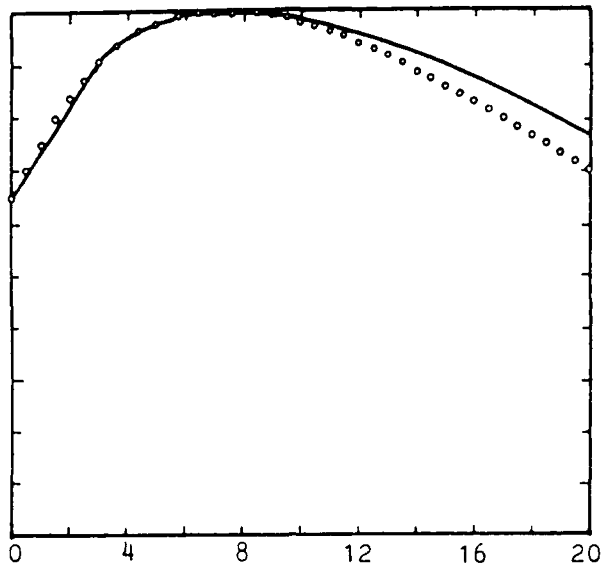


Figure 1b

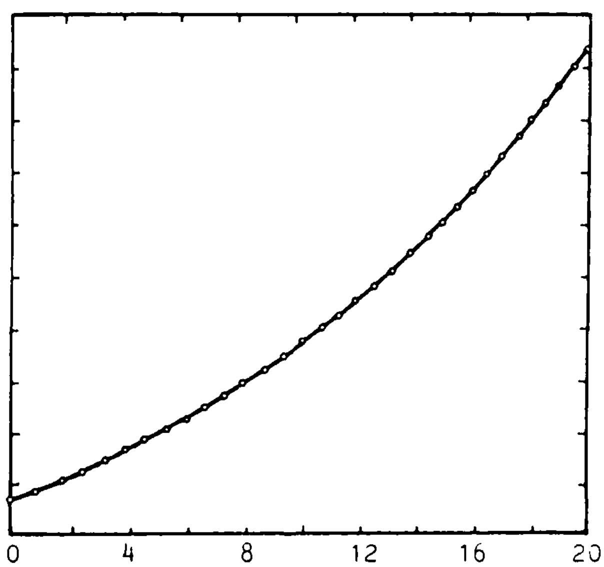
SOLITON WIDTH



DISTANCE

Figure 2a

SOLITON POSITION



DISTANCE

Figure 2b

SOLITON AMPLITUDE

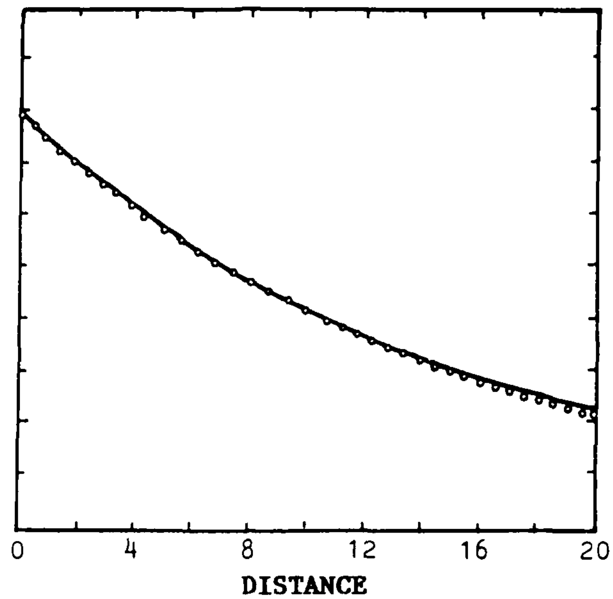


Figure 2c

LOCAL FREQUENCY

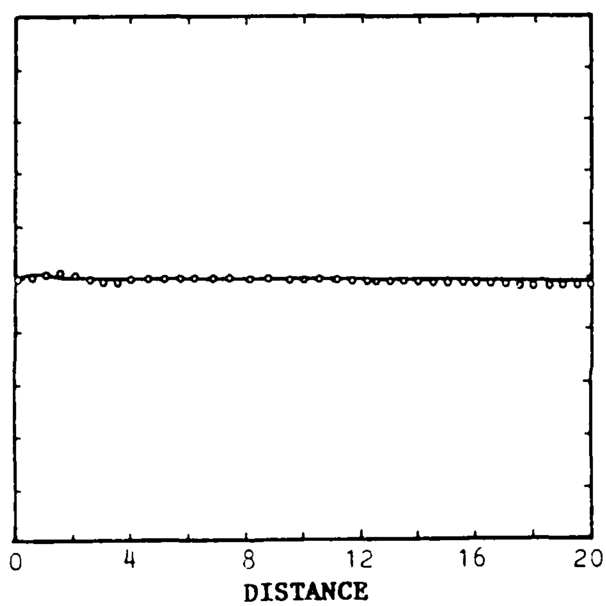


Figure 2d

SOLITON WIDTH

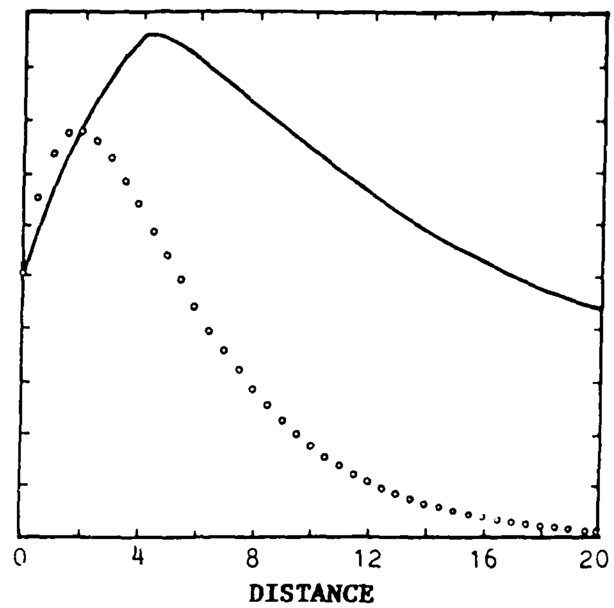


Figure 3a

SOLITON POSITION

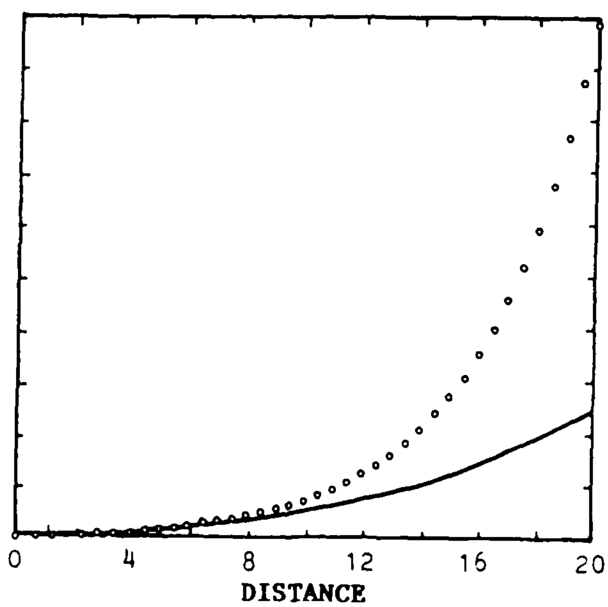


Figure 3b

SOLITON AMPLITUDE

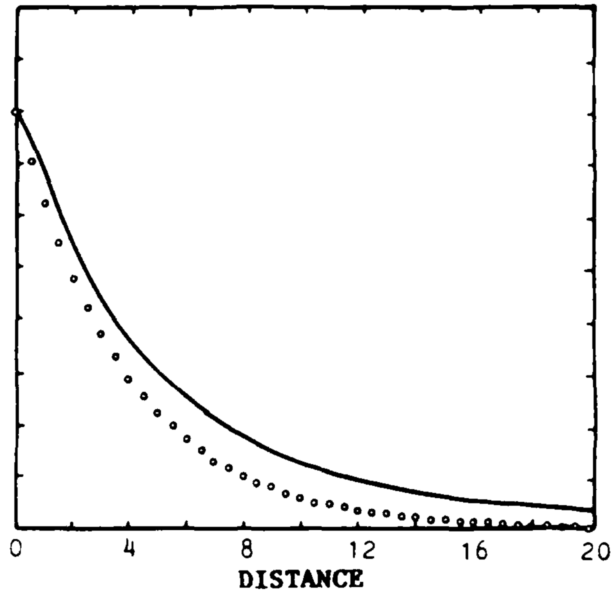


Figure 3c

LOCAL FREQUENCY

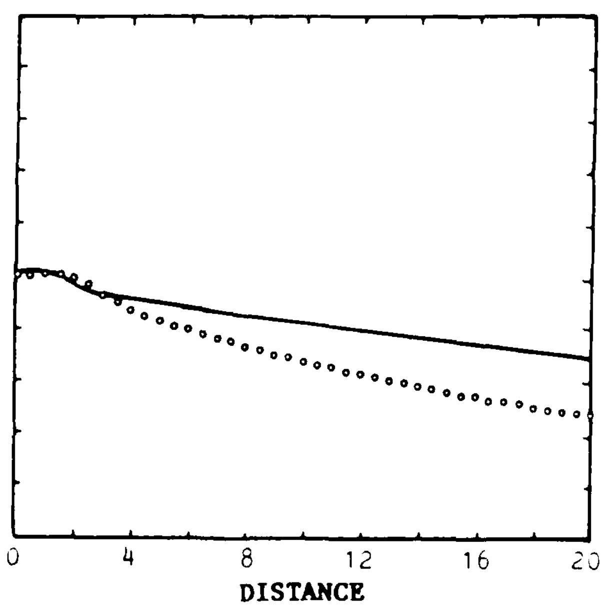


Figure 3d

APPENDIX B

RAMAN SOLITONS IN HOMOGENEOUSLY AND INHOMOGENEOUSLY BROADENED MEDIA

by

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ABSTRACT

We study the propagation of Raman solitons in media with homogeneous and inhomogeneous media. Analytical methods using differential conservation laws for photon energy and momentum currents give good predictions for the rates of soliton narrowing and decay in the hypertransient regime. Our results are verified and extended into the transient regime through numerical studies. Inhomogeneous broadening is found to lead to better soliton stability than homogeneous broadening, and limits on frequency detuning from exact Raman resonance are derived.

I. INTRODUCTION

Solitons in stimulated Raman scattering (SRS) were first observed experimentally in 1983 [1]. Stimulated Raman scattering involves at least two optical beams, one beam called the pump and a second beam called Stokes beam at a frequency lower than the pump. The difference in frequencies is equal or almost equal to the transition frequency of a Raman active medium, through which both beams propagate. The detailed dynamics of scattering depends on the time scale of variation of the optical fields. If this time scale is long compared to the coherence time of the medium (steady state regime) and the medium is in the ground state, the Stokes beam is always amplified and the pump beam is eventually depleted. If the field time scale is comparable to the coherence time (transient regime), the opposite process becomes possible. As the medium becomes excited, photons can be transferred from the Stokes beam back to the pump, reducing the amount of medium excitation and reversing the process of pump depletion [2,3,4].

In the limit where the field time scale is much shorter than the coherence time (hypertransient regime) photons can be transferred freely in both directions. Raman solitons [1,5] are a coherent transient phenomenon in which the opposite processes of pump depletion and amplification occur in a completely balanced way, leaving the medium in the initial unexcited state after the pump pulse has passed. The result is a stable localized non linear wave of excitation of both medium and optical fields, which can be observed experimentally as a pulse of pump radiation travelling through the medium in an envelope of Stokes radiation.

This phenomenon shows very close analogies to the phenomenon of self induced transparency (SIT) [6,7]. In fact an exact formal correspondence can be established between the equations describing the two phenomena. The two levels of the atomic system in SIT correspond to the two levels of the photon system defined by the two beams in SRS, while the electric field in SIT corresponds to the coherent polarization in SRS. Also the roles of spatial and temporal variables are interchanged.

Similar phenomena have been investigated theoretically for coherent two photon propagation [8] and for three level systems, where all transitions are nearly resonant with an optical frequency [9].

Solitons in SRS have been studied theoretically for various model systems [5,10,11]. Strictly speaking they exist only in the hypertransient limit, where coherence decay can be neglected, and the equations for SRS are completely integrable. Their perfect stability implies also, that they cannot be generated other than from optical fields, whose temporal profile closely matches the required form. These are of rather special form which may be difficult to generate and precisely time for the two beams involved, and it seems that mainly for this reason Raman solitons have attracted little attention before.

On the other hand in the presence of coherence decay the equations are no longer exactly integrable. Soliton like excitations exist in the transient and hypertransient regime, which closely correspond to solitons and in many cases converge to solitons in the limit of large gain [1,12,13,14]. Their properties can be analyzed and understood in terms of the integrable system (vanishing line width), and for these reasons we shall use the term "soliton", although they are not solitons in the strict sense of the term.

These solutions can be generated form a large class of initial conditions [1,14,15,16], which includes conditions appropriate for Raman amplifiers with small Stokes seed. The important ingredient here is a rapid phase shift in the injected Stokes beam, which triggers the transient phase wave characteristic for the fully developed soliton. In fact it has been found experimentally [14] and in an independent theoretical study [17] that solitons can even be generated from phase fluctuations in spontaneous scattering. They provide thus a new manifestation of quantum fluctuations on the macroscopic level in addition to the intensity fluctuations found for linear SRS [18]. Soliton like solutions have also exist for more complicated Raman systems including higher order Stokes waves and four wave mixing effects [19].

While coherence decay is thus instrumental in allowing to generate Raman solitons, it also affects the propagation of the fully developed soliton. In particular it has been found that the temporal width of the undepleted pump pulse narrows with increasing propagation distance [[illegible] 14, 15 [illegible]]. If the pump and Stokes beam are not exactly in resonance with the Raman transition, the amplitude of the pump wave [illegible] with increasing propagation distance [14].

It is important to [illegible] quantitative understanding of these effects, both in view of the fundamental importance of this new coherent transient and with respect to possible applications. It has recently been suggested [20] that solitary waves of the type discussed here may be able to accelerate charged particles in a plasma to relativistic velocities. Additional applications may [illegible] optical communications.

In this paper we discuss the effects of coherence relaxation on the propagation of Raman solitons for homogeneously and inhomogeneously broadened Raman media. [illegible] differential conservation laws for energy and momentum, which give formal expression to the balance of physical processes in the system. In the presence of broadening the corresponding quantities are no longer conserved, but satisfy a system of differential equations. If the time dependence of the optical fields is assumed to be of one soliton form with position dependent parameters, a simple closed set of equations for these parameters results, which is exact to lowest order in the broadening. This method is closely related to and has been employed in the context of asymptotic perturbation theory based on the inverse scattering transform (IST) [21,22,23].

In the present context this approach is useful from several points of view. Firstly accurate analytical results are obtained in a simple and transparent way, without having to employ the powerful and mathematically complex tools of IST. Secondly the conservation laws and the derived differential equations have a direct physical interpretation in terms of the perfect or imperfect balance of opposite physical processes (absorption and emission, gain and loss) which governs the situation at hand. Perfect balance is maintained in the absence of coherence decay and results in soliton stability. Coherence decay on the other hand favors certain processes (like loss for the pump field) and leads to a change in soliton parameters. As a result the dynamics of soliton propagation can be understood in physical terms and qualitative predictions can be made even for cases where perturbative methods are no longer adequate.

In section II we give the SRS equations and differential conservation laws for homogeneously broadened media [illegible] soliton solutions for the case of vanishing line width. [illegible] conservation laws are then integrated over time to give equations for the spatial dependence of soliton parameters in the case of non vanishing line width.

In section III we apply the same approach to the case of inhomogeneous broadening for exactly resonant [illegible] we discuss the connection with and main differences [illegible] homogeneously broadened case.

In section IV we present numerical results for the cases discussed in sections II and III [illegible] which broadening cannot be treated as a perturbation.

A summary and [illegible] is given in section V.

II. SOLITON PROPAGATION IN HOMOGENEOUSLY BROADENED MEDIA

The equations for transient [illegible] coherence decay are given by [illegible]

$$Q_{\tau} = -\gamma Q + A_P A_S^* , \qquad (2.1)$$

$$A_{P\chi} = -Q A_S , \qquad (2.2)$$

$$A_{S\chi} = Q^* A_P .$$

Here A_S and A_P are the Stokes and pump electric fields. Q ist the off diagonal matrix element for the molecular Raman transition, and may be considered as an amplitude of coherent excitation of the medium (optical phonon). τ and χ are time like and space like coordinates, which are related to time t and propagation distance z in the laboratory frame by $\tau = t - z/c$ and $\chi = z$. Partial differentiation with respect to these coordinates is indicated by the corresponding subscript. The first term in (2.1) describes collisional coherence decay with decay time $1/\gamma$ where γ is the angular Raman line width (HWHM Lorentzian) in radians per unit of time. Suitable units have been chosen to render all coupling constants equal to unity.

Certain effects are neglected in these equations. In addition to higher order Stokes generation and four wave mixing these are effects of dynamic Stark shift, medium polarization and medium saturation (population of the upper level)[10,18,29]. Their validity is hence limited to sufficiently low intensities.

It is convenient to introduce a Bloch vector S for the optical fields by

$$S_+ = S_1 + i S_2 = 2 A_P A_S^* , \qquad (2.3)$$

$$S_3 = A_P A_P^* - A_S A_S^* ,$$

$$S_0 = A_P A_P^* + A_S A_S^*$$

$$S_1 S_1 + S_2 S_2 + S_3 S_3 = S_+ S_+^* + S_3 S_3 = S_0 S_0 = 1.$$

In terms of S the equations (2.1) and (2.2) can be reformulated as

$$Q_{\tau} = -\gamma Q + 0.5 S_+ , \qquad (2.4)$$

$$S_{+\chi} = 2 Q S_3 , \qquad (2.5)$$

$$S_{3\chi} = -2 \,\mathrm{Real}\,(Q^* S_+) ,$$

$$S_{0\chi} = 0 .$$

The last equation in (2.5) is a differential conservation law expressing conservation of photon number. In the absence of coherence decay (γ=0) equations (2.4) and (2.5) are integrable

[5], and admit an infinite number of additional differential conservation laws. For $\gamma>0$ the corresponding equations are no longer conservation laws, but include dissipative terms.

Solutions for $\gamma = 0$ are found by the method of IST, where a certain function of the physical amplitudes is used as a potential in an associated linear wave equation (see references given above). Two formulations of the initial value problem are possible. In the first formulation the dependent variables are assumed to be given for all positions at some initial time, and their time evolution is sought. In this case the phonon amplitude Q appears as a potential in the associated linear equation [5]. In the second formulation the dependent variables are assumed to be given at all times for some initial position, and their spatial evolution is sought. The soliton potential is now a function of the optical fields [15]. While this formulation directly solves the physical initial value problem, where the optical fields are given at the entrance to the scattering medium, the corresponding conservation laws are more difficult to handle, since they involve non local functions of the fields. We shall therefore use the conservation laws obtained from the first formulation. The first two conservation laws involving the phonon amplitude Q are given by [5,24]:

$$(Q Q^*)_\tau - 0.5\, S_{3\chi} = -2\gamma Q Q^* , \tag{2.6}$$

$$\mathrm{Im}(Q^* Q_\chi)_\tau + \mathrm{Im}(0.5\, S_+ Q^*)_\chi = -2\gamma\, \mathrm{Im}(Q^* Q_\chi) . \tag{2.7}$$

The first equation is a conservation law for polarization energy, balancing the time change of polarization energy density in the medium with the divergence of the energy current carried by the optical fields. For $\gamma=0$ the pump is depleted in space time regions of increasing medium polarization, and depletion is reversed in region of decreasing polarization. For $\gamma>0$ energy is lost to the medium, and the direction of energy flow for the fields depends on the net balance between the rates of polarization change and energy loss. In steady state only the loss term is kept, and the pump is always depleted.

The second equation gives a non trivial result only for the case, where there is a space and time dependent phase difference between fields and medium excitation, for example when the optical fields are not exactly in resonance with the Raman transition. In this case energy and momentum balance is maintained through the generation of an optical phonon wave. Equation (2.7) is a momentum conservation law for this wave, which balances the time change of the momentum density of the phonon wave (first term) with the divergence of the excess momentum current carried by the fields (second term). The latter current is obtained from (2.2) as:

$$\mathrm{Im}(A_{P\chi} A_P^* + A_{S\chi} A_S^*) = \mathrm{Im}(Q^* S_+) . \tag{2.8}$$

In the presence of coherence decay momentum is dissipated in the medium, which is described by the right hand term.

We now consider the one soliton solutions for the case $\gamma = 0$ [16]:

$$Q = \exp(iB)\,\mathrm{sech}(A) \quad , \tag{2.9}$$

$$A_P = \sqrt{\mu_I\,\omega_R}\,\exp(iB)\,\mathrm{sech}(A) \quad , \tag{2.10}$$

$$A_S = \sqrt{\mu_I/\omega_R}\,[\,\omega_R \tanh(A) + i\,\omega_I\,] \quad ;$$

$$S_+ = -2\,\mu_I \exp(iB)\,\mathrm{sech}(A)\,[\,\omega_R \tanh(A) - i\,\omega_I\,] \, , \tag{2.11}$$

$$S_3 = (\,2\,\varrho\,\mathrm{sech}^2(A) - 1\,) \quad ;$$

$$A = \omega_R \tau - \mu_I \chi \;, \quad B = \omega_I \tau - \mu_R \chi \;, \tag{2.12}$$

$$\mu_I = \omega_R/(\,\omega_R^2 + \omega_I^2) \;, \quad \mu_R = \omega_I/(\,\omega_R^2 + \omega_I^2\,) \;.$$

This solution describes a coherent excitation of both medium and fields travelling at a speed v smaller than the speed c of light. Except for a phase factor the phonon amplitude is symmetric about the temporal center (A=0). Hence gain and loss for the optical fields are exactly balanced (see 2.6) and photons are transferred back to the pump pulse in the trailing edge of the soliton. The symmetric shape of the phonon amplitude is in turn caused by a rapid phase shift in the Stokes field at the soliton center. The excitation can be observed experimentally as a localized pulse of pump radiation or as an (infinitely) extended pulse of Stokes radiation with a localized dip in intensity.The temporal width of the excitation is equal to $\Delta\tau = 1/\omega_R$ which defines an intrinsic time scale. The solution will be a valid approximation to (2.1) and (2.2) if $\gamma \ll \omega_R$. The frequency of the pump field is detuned from exact Raman resonance by $\Delta\omega = \omega_I$. If $\omega_I \neq 0$ the optical fields carry a phase wave with local frequency ω_{loc}, whose maximum value at soliton center is equal to $1/\mu_R$:

$$S_+ = |\,S_+|\,\exp(\,i\,\phi\,) \quad , \quad \phi_\tau = \omega_{loc} \, , \tag{2.13}$$

$$\omega_{loc} = \omega_I + \omega_R^2\,\omega_I/(\,\omega_I^2 \cosh^2(A) + \omega_R^2 \sinh^2(A)) \;.$$

At maximal height the pump pulse reaches a fraction ϱ of total intensity. Equations (2.14) summarize the observable parameters of the soliton and their relations:

$$\Delta\tau = 1/\omega_R \quad , \quad \Delta\omega = \omega_I \quad , \tag{2.14}$$

$$\varrho = \mu_I\,\omega_R = 1/(1 + \Delta\omega^2\Delta\tau^2) \, ,$$

$$1/v = 1/c + 1/(\omega_R^2 + \omega_I^2) .$$

The three components of the optical field Bloch vector and the local frequency are shown in figure 1. For sufficiently large negative values of A the solution closely approximates the physical boundary conditions for a medium extending into the positive half plane $\chi > 0$ and optical fields vanishing or assuming their asymptotic values for $\tau < 0$.

In the presence of coherence decay solutions exist which show similar features as the soliton discussed above. In particular localized excitations exist which show reversal of pump depletion [1,14,15,16,17]. If the typical time scale of change for the fields is larger than or comparable to the coherence time (steady state or transient regime), these solutions may appear to be quite different from the soliton solutions discussed above [25]. However if the relative phase between the optical fields varies sufficiently slowly in time, these solutions will show temporal narrowing and eventually approximate the soliton solutions closely, as the width becomes much smaller than the coherence time (hypertransient regime). Since we are primarily interested in the dynamics of this approach to the hypertransient regime, a discussion in terms of the exact soliton solutions for the solvable case is appropriate and useful.

As a result of coherence decay temporal width, position and maximal amplitude will be general functions of propagation distance, and the pulse shapes will no longer be symmetric. It is found that to first order in γ the changes in width, position and amplitude can be calculated without taking into account the changes in pulse shape [21]. We shall calculate these parameters by integrating equations 2.6 and 2.9 over time. This gives ordinary differential equations in χ for the corresponding integrals. The integrals are calculated by using solutions 2.9 and 2.11, however with parameters ω_R and ω_I that are functions of χ . The resulting equations are:

$$(d/d\chi)\ I_k = -2\gamma J_k , \quad k = 1,2 ; \tag{2.15}$$

$$I_1 = \int d\tau\ 0.5[S_3 - S_{3\infty}] = 2 \rho\ \omega_R = 2 \mu_I , \tag{2.16}$$

$$J_1 = \int d\tau\ (Q Q^*) = 2 \mu_I^2\ \omega_R ,$$

$$I_2 = \int d\tau\ \mathrm{Im}(0.5 S_- Q^*) = -2 \mu_R \mu_I = - \mu_R I_1 ,$$

$$J_2 = \int d\tau\ (Q^* Q_\chi) = -2 \mu_R\ \mu_I^2 \omega_R = \mu_R J_1 .$$

The solutions are most easily expressed in terms of the parameters and for = 0:

$$\mu_R(\chi) = \mu_R = \text{const} \quad , \tag{2.17}$$

$$\rho(\chi) = \rho(0)\exp(-L) \quad , \quad L = 4\gamma\mu_R^2\chi \quad ;$$

$$\omega_R^2 = \rho/(1-\rho)/\mu_R^2 \quad , \tag{2.18}$$

$$\omega_I = (1-\rho)/\mu_R \qquad \text{for} \quad \omega_I \neq 0 \quad ;$$

$$\omega_R^2 = \omega_R^2(0) + 4\gamma\chi \qquad \text{for} \quad \omega_I = 0 \quad .$$

The coefficients involving the propagation distance can be expressed in terms of the steady state gain G or the transient gain G' . The transient gain G' is a measure for the spatial change of the soliton fields and is appropriate for a discussion of the internal dynamics of soliton propagation. It is defined from equation (2.12) as $G' = 2\mu_I\chi$. On the other hand for soliton experiments the steady state gain G is more relevant, since its value is limited by processes like higher order Stokes generation to (approximately) $G < 25$. This parameter determines the growth of Stokes intensity for undepleted pump in the steady state regime:

$$I_S = I_S(0)\exp(G) \quad , \qquad G = 2\chi\gamma \quad . \tag{2.19}$$

In terms of G the coefficients in (2.17) and (2.18) are given by:

$$4\gamma\chi = 2\gamma^2 G \quad , \qquad L = 2(\gamma\mu_R)^2 G \quad . \tag{2.20}$$

For exact Raman resonance ($\omega_I = 0$) we obtain for the soliton width from (2.18):

$$\Delta\tau = \Delta\tau(0)/\sqrt{1 + 2(\gamma\Delta\tau(0))^2 G} \tag{2.21}$$

$$\approx \Delta\tau(0)\left[1 - (\gamma\Delta\tau(0))^2 G\right] \qquad \text{for} \quad (\gamma\Delta\tau(0))^2 G \ll 1 \; ,$$

$$\approx 1/(\gamma\sqrt{2G}) \qquad \text{for} \quad (\gamma\Delta\tau(0))^2 G \gg 1 \; .$$

In the limiting case of small gain and width the width is seen to decrease linearly with gain. The corresponding coefficient is proportional to the square of the initial width. In the opposite limit of large gain the width becomes independent of its initial value, and decreases with the inverse square root of the gain. This limit is not actually reached for realistic gain values in the hypertransient regime. Numerical results show however that the soliton width does indeed become almost independent of the initial width at large gain.

For non zero detuning the soliton amplitude ϱ decreases exponentially, while the parameter μ_R, whose inverse is the maximal local frequency of the corresponding phase wave, remains constant. If the soliton width is of the order of the coherence time, and the detuning is much smaller than the line width, we have:

$$\mu_R \approx \Delta\omega \, (\Delta\tau)^2 , \quad L \approx 2G(\Delta\omega \, \Gamma \, (\Delta\tau)^2)^2 ; \tag{2.22}$$

showing that the attenuation coefficient L for the soliton amplitude is proportional to the square of the detuning. This explains the observed rapid decay of Raman solitons for off resonant SRS [14]. The soliton width reaches its minimum for $\varrho = 0.5$ and increases after that point.

III. SOLITON PROPAGATION IN INHOMOGENEOUSLY BROADENED MEDIA

In inhomogeneously broadened media the matrix element Q, which determines the medium polarization at the optical frequencies is obtained as a statistical average over molecular subpopulations, each with different frequency shift Δ from exact Raman resonance in the laboratory frame. The statistical distribution is characterized by the line shape function $\hat{g}(\Delta)$. If we denote the matrix element for the population with frequency shift Δ by Q^Δ, equation 2.4 for the matrix element is replaced by:

$$Q^\Delta_\tau = i\Delta \, Q^\Delta + 0.5 \, S_+ , \tag{3.1}$$

$$Q = \int d\Delta \; \hat{g}(\Delta) \, Q^\Delta . \tag{3.2}$$

We integrate (3.1), substitute the result into (3.2) and obtain:

$$Q^\Delta(\tau) = \int_{-\infty}^{\tau} d\tau' \, \exp[\, i\Delta(\tau - \tau')] \; 0.5 \, S_+(\tau') , \tag{3.3}$$

$$Q(\tau) = \int_{-\infty}^{\tau} d\tau' \, g(\tau - \tau') \; 0.5 \, S_+(\tau') , \tag{3.4}$$

where

$$g(\tau) = \int_{-\infty}^{\infty} d\Delta \, \exp[\, i\Delta\tau\,] \, \hat{g}(\Delta)$$

is the Fourier transform of the spectral line shape function $\hat{g}$ (temporal correlation function).

We note that the case of homogeneous broadening is included in (3.4) with $g(\tau) = \exp(-\gamma\tau)$. This shows that in the absence of medium saturation homogeneous broadening is equivalent to inhomogeneous broadening with Lorentzian line shape. The conservation laws (2.6) and (2.7) require the time derivative of Q, which is obtained from (3.4) as:

$$Q_\tau = 0.5\, S_+ + \int_{-\infty}^{\tau} d\tau'\ \dot{g}(\tau - \tau')\ 0.5\, S_+(\tau') \qquad (3.5)$$
$$= Q^{(0)} + Q^{(1)} ,$$

where $\dot{g}$ is the derivative of g. The last term in (3.5) is the correction from coherence decay to the exactly solvable case. We shall discuss here only the case of exact resonance at line center, and of a symmetric line shape. In this case Q is real, and we obtain from the first conservation law in analogy with (2.6):

$$(Q\ Q)_\tau + 0.5\, S_{3\chi} = 2\ (\,Q^{(1)}_\tau\ Q\,) \quad . \qquad (3.6)$$

The correction term is calculated to leading order in the line width by expanding the correlation function g:

$$Q^{(1)}_\tau = \dot{g}(0)\ Q^{(0)} + \ddot{g}(0) \int_{-\infty}^{\tau} d\tau'\ \ Q^{(0)}(\tau') \quad + \ldots \quad . \qquad (3.7)$$

Here $Q^{(0)}$ is the matrix element for vanishing line width. For homogeneous broadening the first term gives a non vanishing result: $\dot{g}(0) = -\gamma$, due to the weak decrease of the Lorentzian spectrum at large frequencies. For inhomogeneous broadening on the other hand with a Gaussian line shape or other shape with sufficiently fast decrease at large frequencies the first term vanishes, and the leading contribution is of second order in the line width. In this case we obtain from (3.6) the differential equation:

$$(d/d\chi)\ I_1 = 2\ \ddot{g}(0)\ K_1 \quad , \qquad (3.8)$$

$$I_1 = \int d\tau\ \ Q\ Q \quad = 2/\omega_R \quad , \qquad (3.9)$$

$$K_1 = \int d\tau\ \ Q^{(0)}(\tau) \int_{-\infty}^{\tau} d\tau'\ \ Q^{(0)}(\tau') \quad = 0.5\ \pi^2 \omega_R^4 \quad .$$

For the case of a rectangular spectrum with half width γ the solution is:

$$\omega_R^3(\chi) = \omega_R^3(0) + 0.5\ (\,\gamma\pi\,)^2 \chi \quad . \qquad (3.10)$$

The relation between propagation distance and steady state gain is in this case:

$$G = 2\pi\ g(0)\chi \quad = \quad \pi\chi / \gamma \quad , \qquad (3.11)$$

giving the final result for the soliton width as a function of gain:

$$\Delta\tau = \Delta\tau(0)\ (\ 1 + 0.5\,\pi\,(\ (\Gamma\Delta\tau(0))^3\ G\ )^{(-1/3)} \tag{3.12}$$

$$\approx \Delta\tau(0)\ (\ 1 - (\pi/6)\ (\ \Gamma\Delta\tau(0))^3\ G\ )\ .$$

The rate of decrease is now proportional to the third power of the initial width, and the small gain limit considered in the second line of (3.12) above is the only realistic case.

We have not studied the off resonant case so far. Numerical results indicate, that the behavior can be quite complex in this situation. For sufficiently large detuning width and amplitude can decrease and increase alternatively, and strong pulse deformations may develop. We shall discuss these results in the following section IV.

IV. NUMERICAL RESULTS

The transient SRS equations (2.4) and (2.5) were solved numerically with the one soliton form (2.11) for the optical fields as functions of time as initial condition at zero propagation distance. The soliton amplitude and width were determined for the numerical solutions from the maximal value of S and from the integral I (equation(2.16). For inhomogeneous broadening a rectangular line shape was chosen for numerical convenience, and the line was sampled at a finite number of equally spaced points.

Figure 2 compares numerical (circles) and analytical (solid lines) results in the hypertransient regime and the resonant case for homogeneous (lower curves) and inhomogeneous broadening (upper curves). The agreement is excellent in the latter case, and better than 10% for the homogeneous case. In this and all subsequent cases the time scale is fixed by choosing the initial soliton width as the unit of time. The line width is 0.1 for the homogeneous and 0.1571 for the inhomogeneous case, which results in the same gain per propagation distance and the same coherence time for both cases. The soliton width is hence only 10% of the coherence time. At a gain of 20 the homogeneous soliton has narrowed by about 15%, while the inhomogeneous soliton has narrowed only by about 3%.

Figures 3 and 4 compare numerical (circles) and analytical (solid lines) results for soliton width and relative amplitude in the off resonant case and for homogeneous broadening. Initial soliton width and line width are as before and correspond to the hypertransient regime. The values chosen for detuning are 0.1, 0.5 and 1.1. These values correspond to the lower, middle and upper set of curves in figure 3, and to the upper, middle and lower set in figure 4. As predicted the soliton width (figure 3) is larger for larger detuning. For the first two cases is actually larger than predicted, with a predictive accuracy of about 10% . Somewhat larger discrepancies occur for a detuning of 1.1 . Here the soliton width is predicted to increase, while the actual width almost remains constant. A similar behavior is found for the soliton amplitude (figure 4). Larger discrepancies occur again for a detuning of 1.1 , where the actual amplitude is larger than predicted. These discrepancies are caused by the behavior of the phonon wave. In particular the local frequency (see equation (2.13)) begins to show strong deviations from the one soliton form (2.13) at larger detunings, including an almost linear frequency chirp with increased detuning at the trailing edge of the soliton.

The effects of detuning are much more pronounced for inhomogeneous broadening, especially if the detuning is comparable to or exceeds the line width. This is plausible in view of the fact that the weight of higher frequency components is much smaller in the inhomogeneous case than in the homogeneous case. A thorough physical analysis and analytical results remain to be given. Figure 5 shows numerical results for soliton width (solid lines) and amplitude (dotted lines) for a rectangular line with a half width of 0.1571 and an initial soliton width of 1.0 (hypertransient regime). The values for detuning are 0.05, 0.1 and 0.2. At given propagation distance the soliton width increases and the amplitude decreases with increasing detuning. For small detuning (0.05) the width decreases as a function of distance, while it increases initially for large detuning (0.2) and has a maximum at a gain of about 12. At about the same gain the soliton amplitude reaches a minimum. This behavior is in strong contrast to the homogeneous case.

Soliton narrowing is much stronger in the transient regime, where the temporal soliton width is comparable to the coherence time. In the analytical treatment above we found that the narrowing rate per gain unit is proportional to the second power of the line width for homogeneous and to the third power for inhomogeneous broadening. Figures 6 and 7 show numerical results for width and amplitude for homogeneous and inhomogeneous broadening. The initial soliton width is 1.0 and the line width is 1.0 for homogeneous and 1.571 for inhomogeneous broadening. The values for detuning are 0.05, 0.1 and 0.2.

For homogeneous broadening (figure 6) a detuning of 0.05 or 5% of the line width must be considered a limit for practical applications to pulse narrowing. In this case the soliton decays to about 90% at a gain of 20 and narrows to a temporal width of 20% of its initial value. At 10% detuning the 90% amplitude level is reached already at a gain of 7, and the soliton width is 40% of the initial value. It is interesting to note that the initial rate of soliton decay is much less than in the hypertransient regime, where exponential decay is found.

Inhomogeneous broadening (figure 7) leads to better stability for solitons at small detuning than homogeneous broadening. At a detuning of 0.05 or 3% the soliton amplitude reaches a minimum of 90% at a gain of 12, and increases after that point. The narrowing is almost the same as in the homogeneous case, with a larger rate of narrowing at small gain. At a detuning of 0.1 or 6% the soliton amplitude stabilizes at 70% . For larger detuning (0.2 or 13%) an intermediate maximum in amplitude occurs at a gain of [illegible] and the width begins to increase rapidly. At this point long range oscillations begin to develop in the trailing edge of the soliton.

Temporal pulse shapes for the pump beam are shown in figures 8 and 9. Figure 8 shows homogeneous broadening for a detuning of 0.0 (8.a), 0.05 (8.b), 0.1 (8.c) and 0.2 (8.d). Figure 9 shows inhomogeneous broadening for a detuning of 0.0 (9.a), 0.05 (9.b), 0.1 (9.c) and 0.2 (9.d). Strong oscillations are seen to occur in the trailing edge for inhomogeneous broadening. These fade away at larger gain for zero and small detuning (9.a and 9.b), but persist for larger detuning (9.c and 9.d).

V. SUMMARY

Raman solitons correspond experimentally to short localized pulses of pump radiation in an envelope of Stokes radiation. They are sustained by balanced processes of absorbtion and emission. In the presence of coherence decay pulse narrowing and decay occur. Decay occurs only if the two optical fields are not exactly on Raman resonance ("detuning").

We have studied these phenomena both analytically and numerically in the hypertransient regime, and numerically in the transient regime. The cases of both homogeneous and inhomogeneous broadening were studied.

Our analytical treatment was based on differential conservation laws for photon energy and momentum. These conservation laws give mathematical expression to the balance of physical processes and the effects of broadening on this balance. For the off resonant case (frequency detuning) the occurrence of an optical phonon wave is seen to maintain energy and momentum conservation, and to have considerable influence on soliton stability.

In the transient regime we find a detuning of [illegible] (of the Raman line width) to represent a practical limit for the achievement of both appreciable narrowing and good soliton stability. Solitons are more stable for inhomogeneous broadening, however oscillations occur in the trailing edge of the pump pulse and persist for larger detunings.

The strong influence of detuning with only a few percent of the Raman line width will not have severe consequences for media with broad Raman line. For media with narrow lines however the frequency stability of the pump laser may be in this range. Our treatment assumed monochromatic optical beams with fixed frequency detuning. It is possible however that the phase fluctuations associated with the non vanishing laser line width have a similar influence on soliton stability. Recent experimental observations support this possibility [26].

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REFERENCES

1. K.J. Druhl, R.G. Wenzel, and J.L. Carlsten: "Observation of solitons in stimulated Raman scattering", Phys. Rev. Lett. 51, 1171-1176 (1983);

2. G.I. Kachen and W.H. Lowdermilk: "Self-induced gain and loss modulation in coherent, transient Raman pulse propagation", Phys. Rev. A14, 1472-1474 (1976);

3. N. Tan-no, T. Shirahata, K. Yokoto, and H. Inaba: "Coherent transient effect in Raman pulse propagation", Phys. Rev. A 12, 159-168 (1975);

4. J.N. Elgin and T.B. O'Hare: "Saturation effects in transient stimulated Raman scattering", J. Phys. B: Atom. Molec. Phys. 12, 159-168 (1979);

5. F.Y.F. Chu and A.C. Scott: "Inverse scattering transform for wave-wave scattering", Phys. Rev. A 12, 2060-2064 (1975);

6. S.L. McCall and E.L. Hahn: "Self-induced transparency", Phys. Rev. 183, 457-485 (1969);

7. M.J. Ablowitz, D.J. Kaup, and A.C. Newell: "Coherent pulse propagation: A dispersive, irreversible phenomenon", J. Math. Phys. 15, 1852-1858 (1974);

8. N. Tan-no and Y. Higuchi: "Solitary wave solutions in coherent two-photon pulse propagation", Phys. Rev. A 16, 2181-2183 (1977);

9. M.J. Konopnicki and J.H. Eberly: "Simultaneous propagation of short different-wavelength optical pulses", Phys. Rev. A 24, 2567-2583 (1981);

10. D.J. Kaup: "The method of solution for stimulated Raman scattering and two-photon propagation", Physica 6D, 143-154 (1983);

11. H. Steudel: "Solitons in stimulated Raman scattering and resonant two-photon propagation", Physica 6D, 155-178 (1983);

12. K.J. Druhl, J.L. Carlsten, and R.G. Wenzel: "Aspects of soliton propagation in stimulated Raman scattering", J. Stat. Phys. 39, 615-620 (1985);

13. K.J. Druhl and G. Alsing: "Effect of coherence relaxation on the propagation of optical solitons: An analytical and numerical case study on asymptotic perturbation theory", Physica 20D, 429-434 (1986);

14. R.G. Wenzel, J.L. Carlsten, and K.J. Druhl: "Soliton experiments in stimulated Raman scattering", J. Stat. Phys. 39, 621-632 (1985);

15. D.J. Kaup: "Creation of a soliton out of dissipation", Physica 19D, 125-134 (1986);

16. H. Steudel: "Stimulated Raman scattering with an initial phase shift: The pre-stage of a soliton", Opt. Commun. 57, 285-289 (1986);

17. J.C. Englund and C.M. Bowden: "Spontaneous generation of Raman solitons from quantum noise", Phys. Rev. Lett. 57, 2661-2663 (1986);

18. M.G. Raymer, I.A. Walmsley, J. Mostowski and B. Sobolewska "Quantum theory of Stokes pulse energy fluctuations", J. Opt. Soc. Am. B 1, 547 (1984);

19. J.R. Ackerhalt and P.W. Milonni: "Solitons and four-wave mixing", Phys. Rev. A 33, 3185-3198 (1986);

20. C.J. McKinstrie and D.F. DuBois: "Relativistic solitary-wave solutions of the beat-wave equations", Phys. Rev. Lett. 57, 2022-2025 (1986);

21. D.J. Kaup: "A perturbation expansion for the Zakharov-Shabat inverse scattering transform", SIAM J. Appl. Math. 31, 121-133 (1976);

22. D.J. Kaup and A.C. Newell: "Solitons as particles, oscillators, and in slowly changing media: A singular perturbation theory", Proc. R. Soc. Lond. A 361, 413-446 (1978);

23. V.I. Karpman: "Soliton evolution in the presence of perturbation", Physica Scripta 20, 462-478 (1979);

24. G.L. Lamb, Jr.: "Elements of soliton Theory", John Wiley and Sons, Inc., New York, 1980;

25. S. Elyutin and J.N. Elgin: "Comment on recent observation of solitons in stimulated Raman scattering", Opt. Commun. 60, 104-106 (1986);

26. J.L. Carlsten: personal communication, Department of Physics, Montana State University.

FIGURE CAPTIONS

Figure 1 The three components of the optical field B [illegible] for first and second component (dashed and chain dotted lines), third component (solid line) and the local frequency (dotted line) for a Raman soliton with detuning of [illegible] are shown. The third vector component corresponds to the pump intensity. The scale for the local frequency is reduced by a factor of [illegible].

Figure 2 The soliton width in the resonant case is shown as a function of gain [illegible] for homogeneous (lower curves, line width [illegible]) and inhomogeneous broadening (upper curves, line width 0.1571) in the hypertransient regime. Frequencies are in units radians per time unit, where the time unit is the initial soliton width. Solid lines are analytical results and circles are numerical values.

Figure 3 The soliton width in the off resonant case is shown for homogeneous broadening in the hypertransient regime. Solid lines are analytical results and circles are numerical values. Values for detuning are 0.1 (lower curves), 0.5 (middle curves) and 1.1 (upper curves). The Raman line width is 0.1. Frequencies are in units radians per time unit, where the time unit is the initial soliton width.

Figure 4 The soliton amplitude in the off resonant case is shown for homogeneous broadening in the hypertransient regime. Solid lines are analytical results and circles are numerical values. Values for detuning are 0.1 (upper curves), 0.5 (middle curves) and 1.1 (lower curves). The Raman line width is 0.1. Frequencies are in units radians per time unit, where the time unit is the initial soliton width.

Figure 5 Numerical results for soliton width and amplitude in the off resonant case are shown for inhomogeneous broadening in the hypertransient regime. The Raman line width is 0.1571 and the detuning is 0.05, 0.1 and 0.2. In this sequence of values the lower, middle and upper solid lines are the width, while the upper, middle and lower chain dotted lines are the soliton amplitude. Frequencies are in units radians per time unit, where the time unit is the initial soliton width.

Figure 6 Numerical results for soliton width and amplitude in the off resonant case are shown for homogeneous broadening in the transient regime. The Raman line width is [illegible] and the detuning is 0.05, 0.1 and 0.2. In this sequence of values the lower, middle and upper solid lines are the width, while the upper, middle and lower chain dotted lines are the soliton amplitude. Frequencies are in units radians per time unit, where the time unit is the initial soliton width.

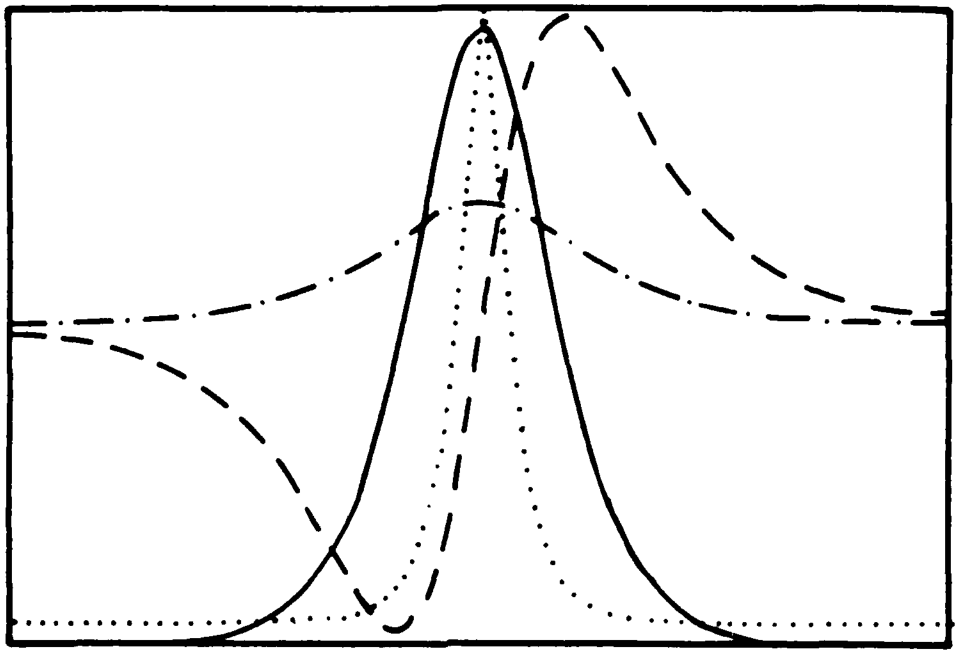
Figure 7 Numerical results for soliton width and amplitude in the off resonant case are shown for inhomogeneous broadening in the transient regime. The Raman line width is 1.571 and the detuning is 0.05, 0.1 and 0.2. In this sequence of values the lower, middle and upper solid lines are the width, while the upper, middle and lower chain dotted lines are the soliton amplitude. Frequencies are in units radians per time unit, where the time unit is the initial soliton width.

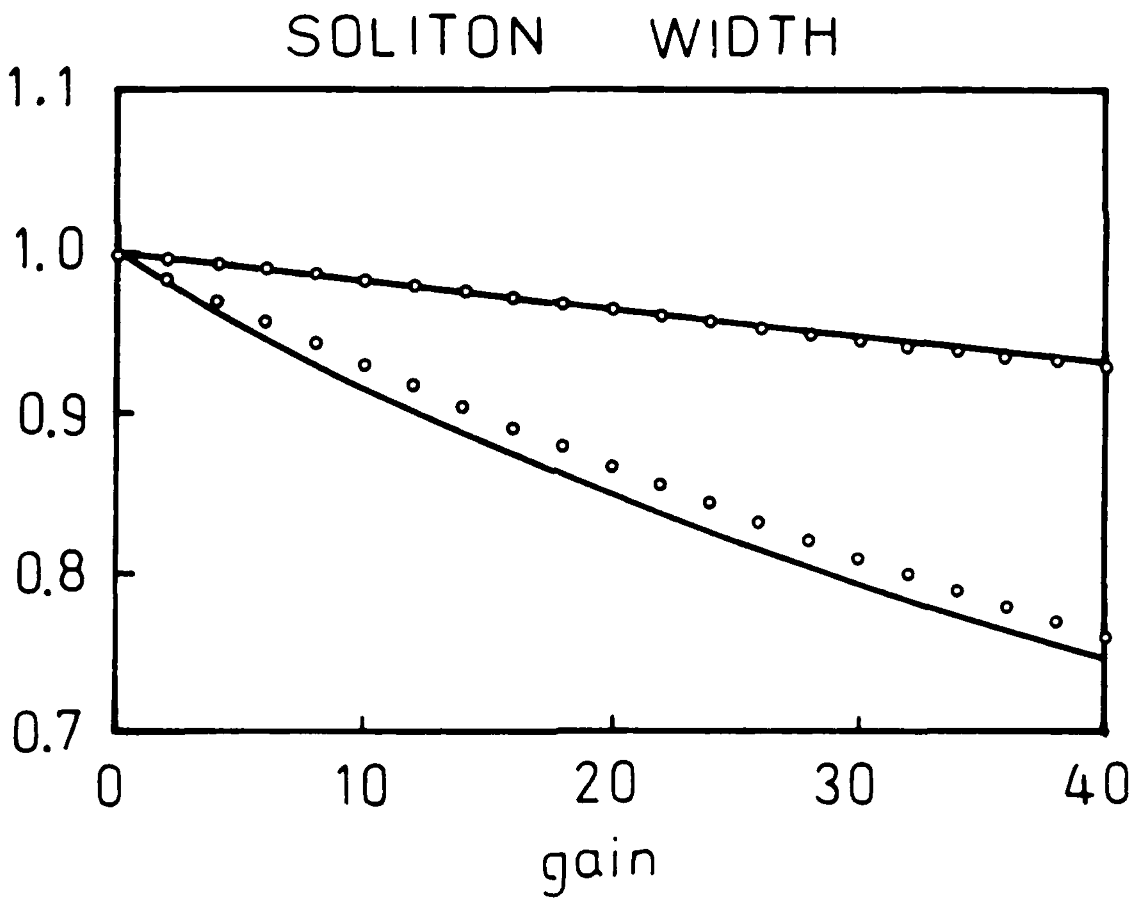
Figures 8a and 8b The temporal pump pulse for homogeneous broadening in the transient regime is shown for a detuning of 0.0 (8a) and 0.05 (8b). Pulses are shown for a gain increment of 2, beginning with the initial pulse in the upper right part.

Figures 8c and 8d The temporal pump pulse for homogeneous broadening in the transient regime is shown for a detuning of 0.1 (8c) and 0.2 (8d). Pulses are shown for a gain increment of 2, beginning with the initial pulse in the upper right part.

Figures 9a and 9b The temporal pump pulse for homogeneous broadening in the transient regime is shown for a detuning of 0.0 (9a) and 0.05 (9b). Pulses are shown for a gain increment of 2, beginning with the initial pulse in the upper right part.

Figures 9c and 9d The temporal pump pulse for homogeneous broadening in the transient regime is shown for a detuning of 0.1 (9c) and 0.2 (9d). Pulses are shown for a gain increment of 2, beginning with the initial pulse in the upper right part.





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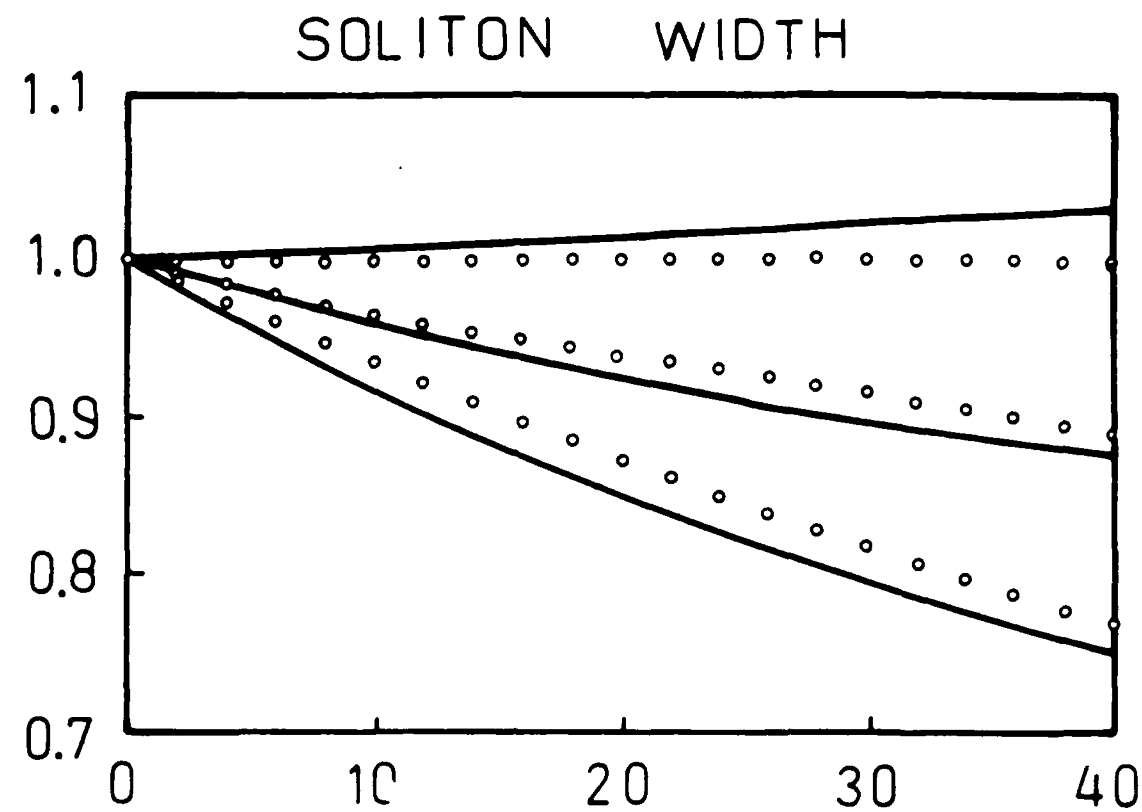

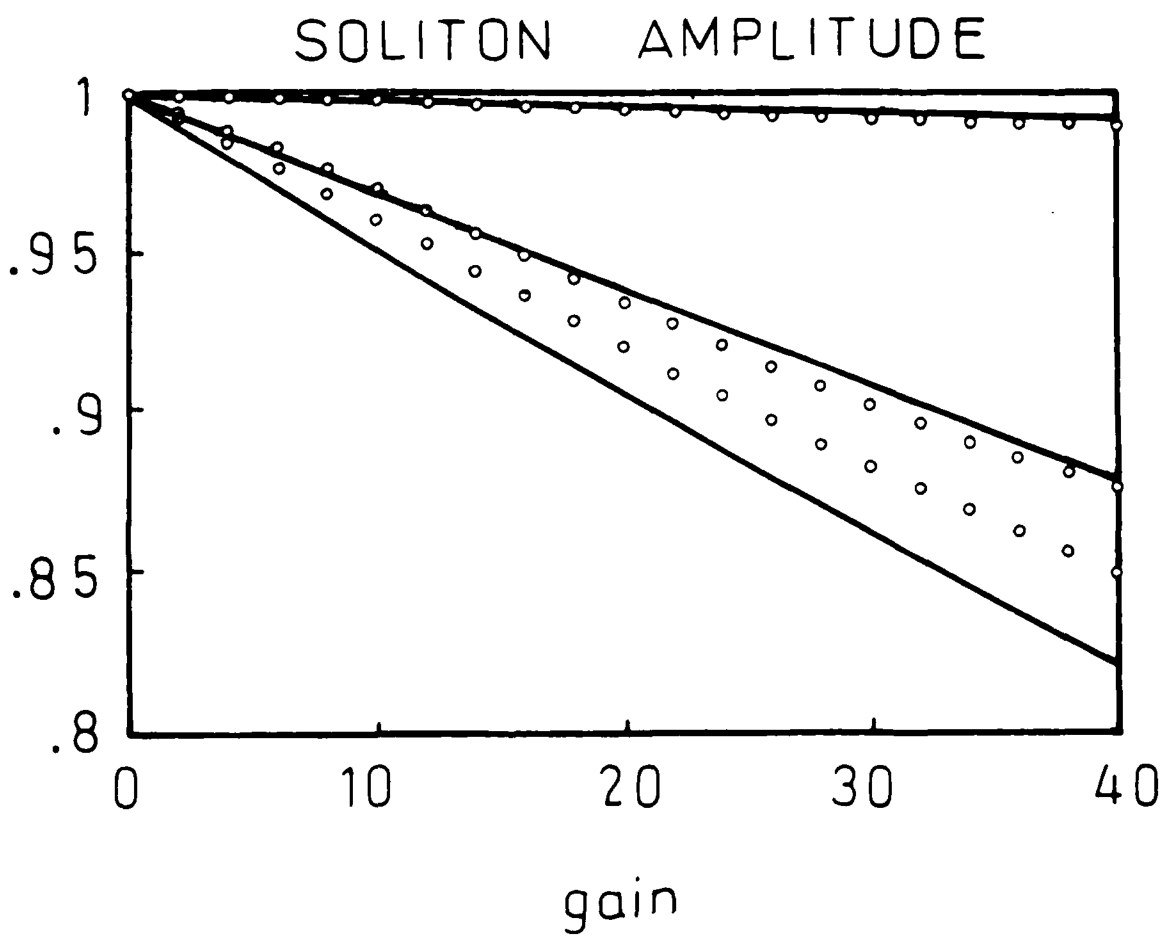

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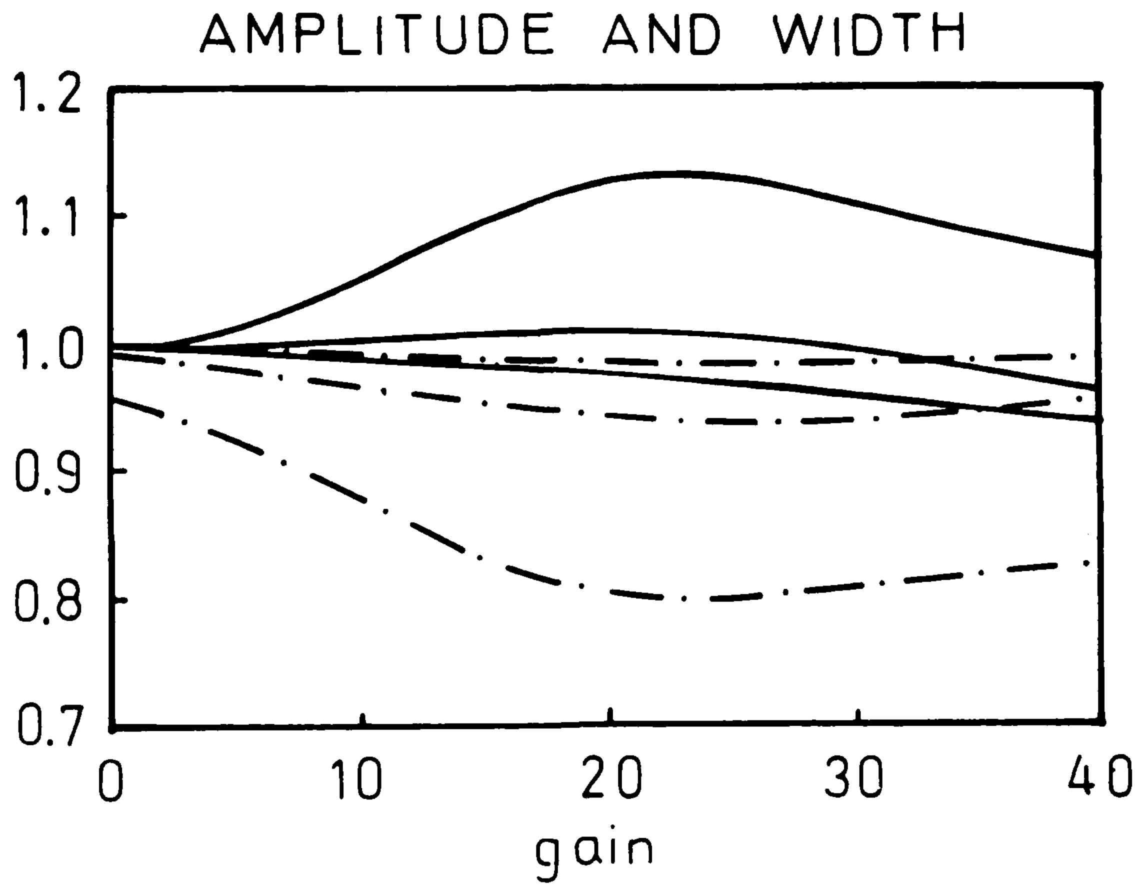
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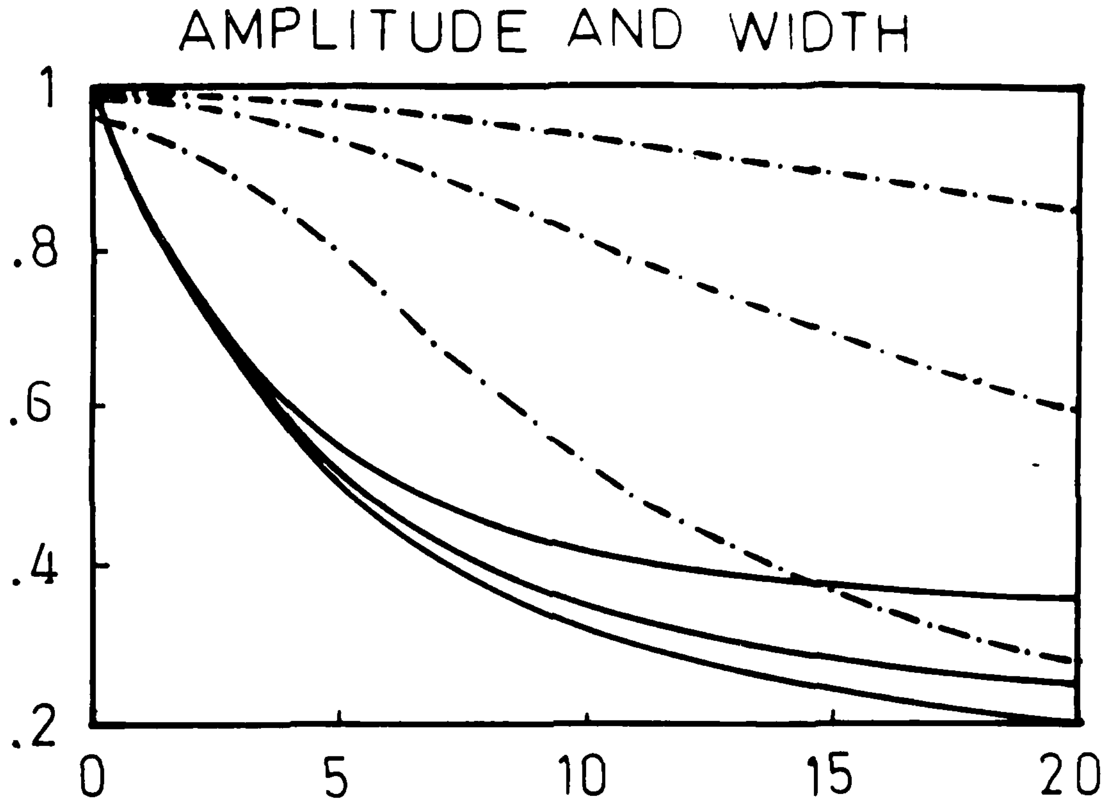


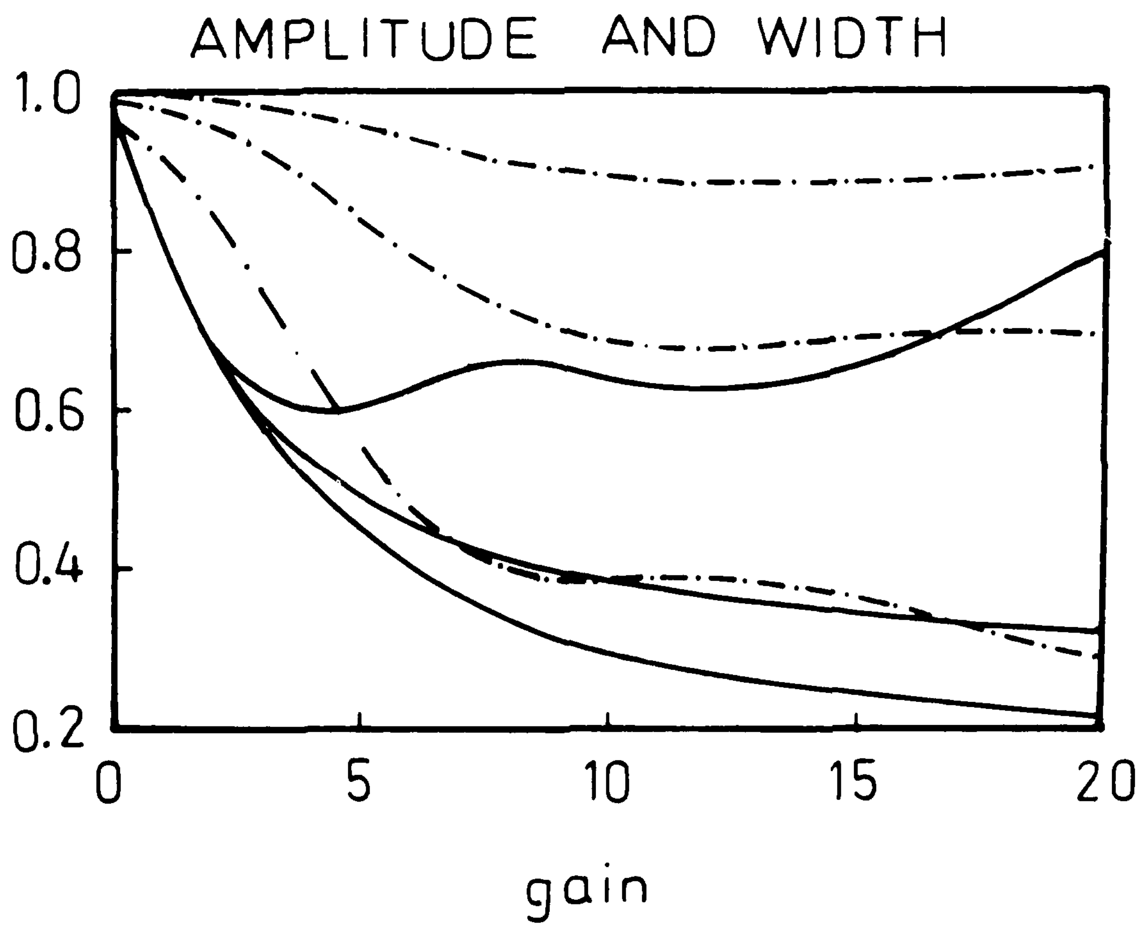
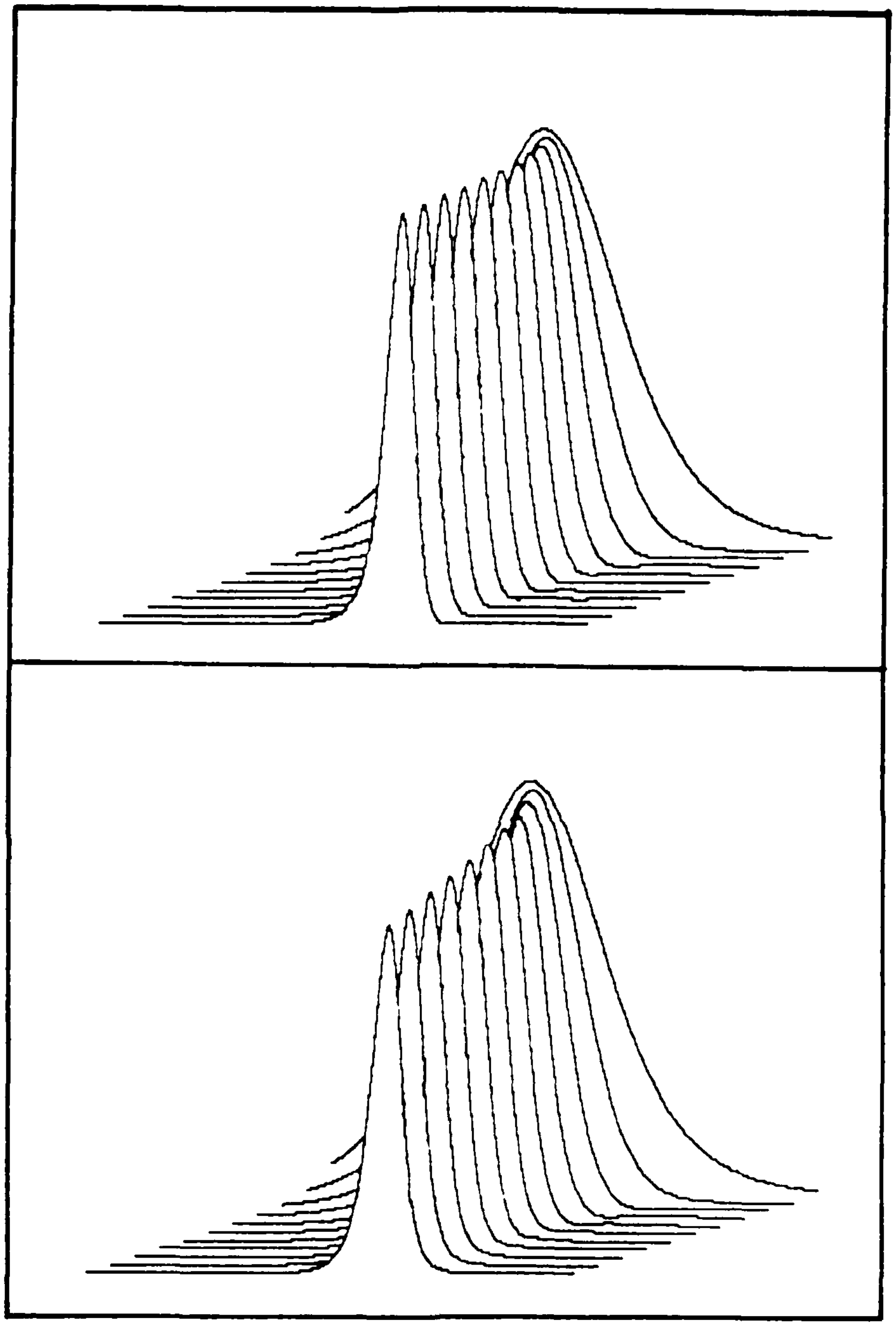


Figure 6 (top) and Figure 7 (bottom)



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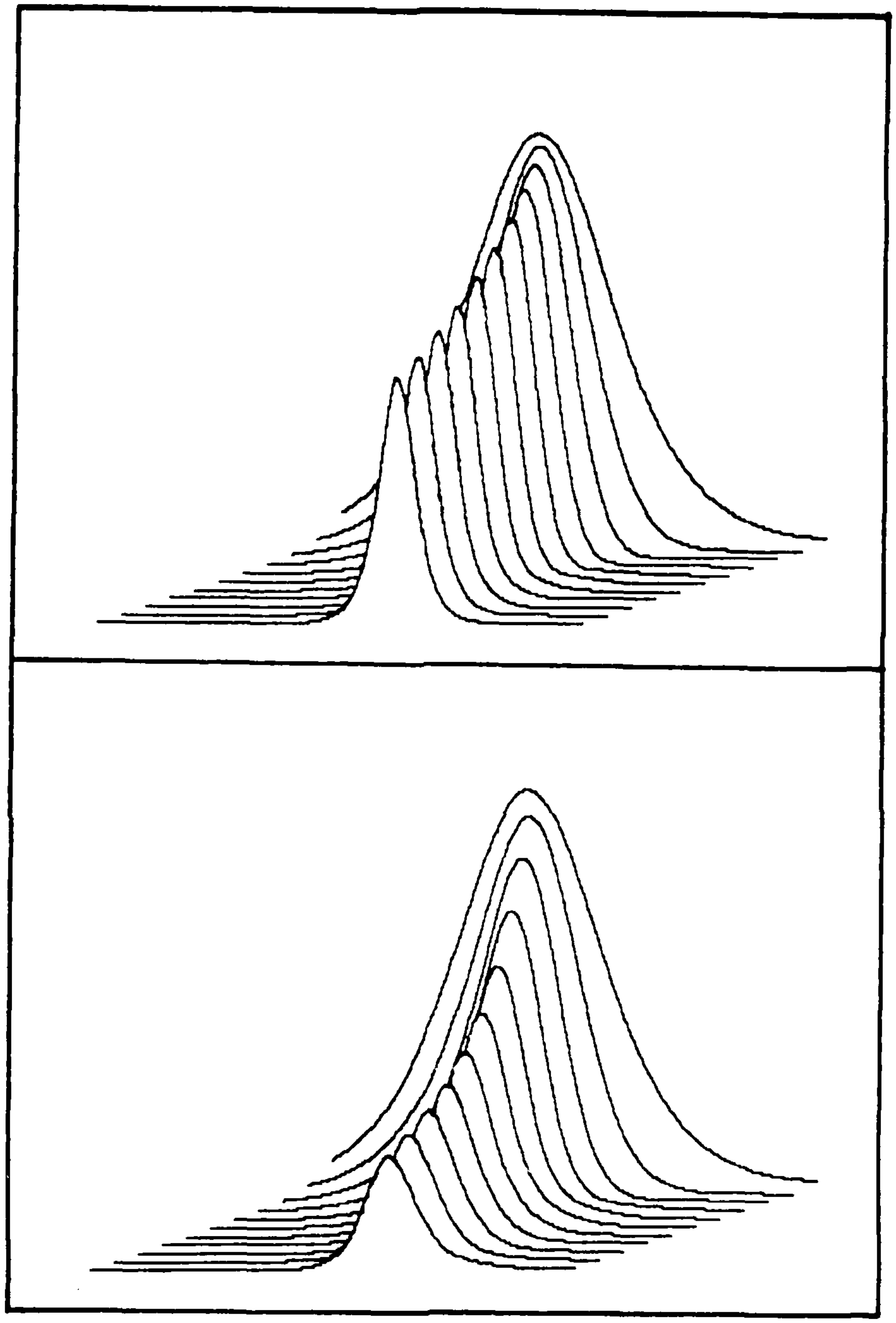

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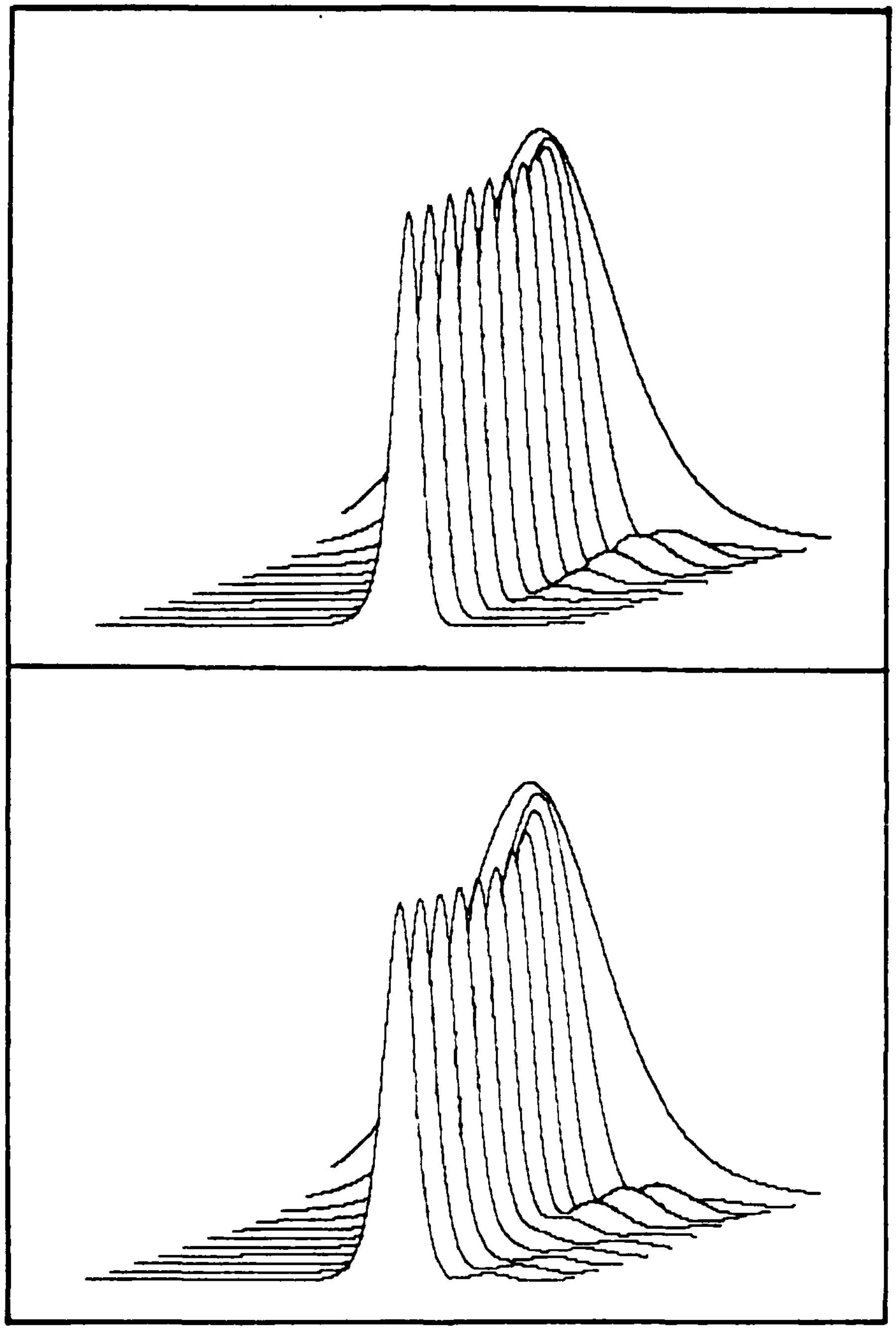

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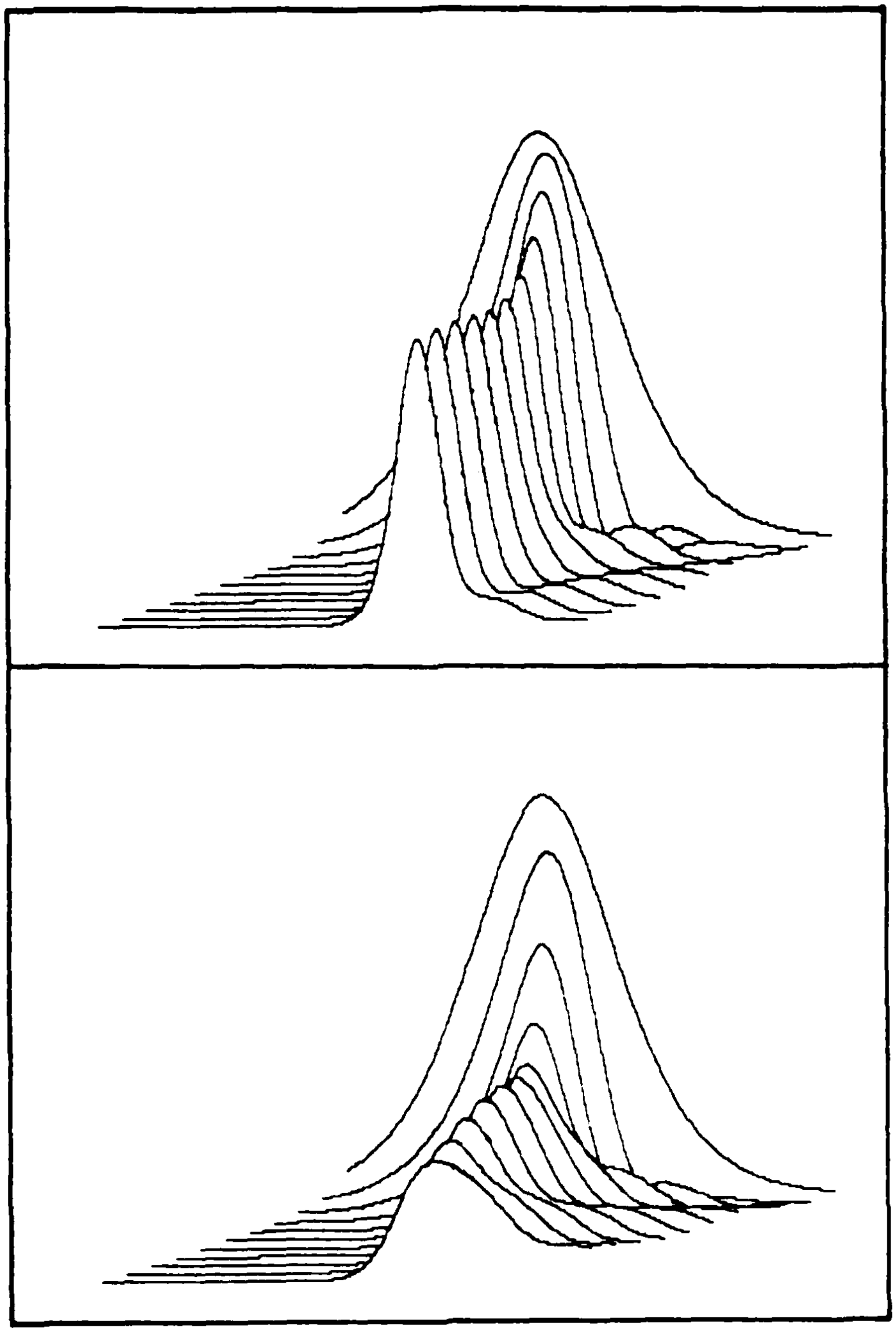

APPENDIX C

Effect of Medium Saturation and Coherence Relaxation on the Propagation of Raman Solitons

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ABSTRACT

The effects of medium saturation and homogeneous broadening on the propagation of solitons in stimulated Raman scattering are discussed. The general problem including Stark shift and medium saturation, but without broadening has been solved by Kaup and Steudel using an extension of the AKNS method. For the limiting case of negligible Stark shift, we obtain first order equations for the soliton width in the presence of homogeneous broadening. We use these asymptotic solutions to discuss the effect of medium saturation on pulse narrowing and compare them to a numerical analysis.

In stimulated Raman scattering (SRS) energy is exchanged between two laser beams at different frequencies by nonlinear interaction in a suitable medium. Energy is normally transferred from the high frequency beam (pump) to the low frequency beam (stokes). In both numerical [1] and laboratory [2,3] experiments, solitonlike excitations have been created by a sudden phase change of 180 degrees in the injected stokes beam. In the presence of collisional coherence decay in the medium and for exact resonance, the pulse shows the features of temporal narrowing and acceleration in the laboratory frame [1,3,4].

The effect of coherence decay on the propagation of Raman solitons has already been studied both analytically and numerically using asymptotic perturbation theory [1,3,4]. In these studies however, certain effects like medium saturation or dynamic Stark shift were neglected.

In this paper, we discuss how medium saturation affects temporal pulse narrowing, using a perturbative method which is based on the energy conservation law for medium and optical fields [5].

The general problem of soliton propagation in stimulated Raman scattering (SRS), including Stark shift and medium saturation, has been solved by Kaup [6] and Steudel [7] using an extension of the AKNS method [8]. In the presence of collisional coherence decay and for the limiting case of negligible Stark shift, Kaup's and Steudel's equations [6,7] for transient SRS become:

$$(r_+)_T = -\gamma r_+ - r_3 s_+ \tag{1.1}$$

$$(r_3)_T = r_+ s_+ \tag{1.2}$$

$$(s_+)_X = s_3 r_+ \tag{1.3}$$

$$(s_3)_X = - r_+ s_+ \tag{1.4}$$

in which r_+ is proportional to the medium polarization induced by the optical fields, r_3 is proportional to the population difference between the two levels of the Raman transition, s_3 is proportional to the difference of photon current densities between the pump and the stokes beams and s_+ is proportional to the product of the stokes electric field and the complex conjugate of the pump electric field. T and X are time and space like coordinates in a retarded time frame and the subscripts indicate partial differentiation with respect to these coordinates. Convenient units have been chosen so that all coupling constants are equal to 1 and the term $-\gamma r_+$ describes collisional coherence decay with coherence time $1/\gamma$, where γ is the angular Raman line width. We have not considered population relaxation here, which we assume to occur on a time scale much longer than the coherence time.

In the hypertransient regime $((r_+)_T \gg \gamma r_+)$, the physical quantities vary on a time scale much shorter than the dephasing time $1/\gamma$. In the limit $\gamma=0$, equations (1.1) to (1.4) are integrable and exact solutions can be found using an extension of the inverse spectral transform [6,7]. For the special case of purely imaginary eigenvalues, the one-soliton solutions to equations (1.1) to (1.4) are:

$$r_+ = -(\eta_1^2-1/4)^{1/2}\cosh(2\mu)/((\eta_1^2-1/4)\cosh^2(2\mu)+1/4) \qquad (2.1)$$

$$r_3 = -1 + (1/2)/((\eta_1^2-1/4)\cosh^2(2\mu)+1/4) \qquad (2.2)$$

$$s_+ = (2\eta_1(\eta_1^2-1/4)^{1/2}\sinh(2\mu))/((\eta_1^2-1/4)\cosh^2(2\mu)+1/4) \qquad (2.3)$$

$$s_3 = -1 + 2\eta_1^2/((\eta_1^2-1/4)\cosh^2(2\mu)+1/4) \qquad (2.4)$$

where η_1 is the imaginary part of the eigenvalue and $\mu=\eta_1 T - X/4\eta_1$.

In this case, the optical fields are exactly in resonance with the Raman transition. In the following we discuss the physical processes underlying soliton propagation for different values of η_1. In the leading edge of the pump pulse ($\mu>0$), s_+ is in phase with the induced polarization r_+ in the medium, leading to a decrease in s_3 (stokes scattering). In the trailing edge ($\mu<0$), however, s_+ has changed sign and s_3 increases now with propagation distance. This means that the stokes beam experiences loss with corresponding gain for the pump beam (antistokes scattering). By the same mechanism the medium is excited in the leading edge and deexcited in the trailing edge. Both processes (loss and gain, excitation and deexcitation) are perfectly balanced, as is evident from the symmetric soliton shape.

Two limiting cases are of interest:

a) when η_1 becomes large ($\eta_1 \rightarrow \infty$), the above analytical solutions are equivalent to the one soliton solutions found by Chu and Scott [9] for the case in which medium saturation is neglected ($r_3 \rightarrow -1$); in this case, the temporal width of the soliton is much smaller than the Rabi time, during which population inversion occurs, and the gain-loss reversal between the stokes and the pump beams occurs very rapidly;

b) when η_1 approaches 1/2, the rise time of s_3 becomes comparable to the Rabi time leading to a population inversion for the medium. The off-diagonal matrix element r_+ is correspondingly small, leading to a more gradual change in the field variables. As a consequence, a wide temporal plateau is formed.

In the transient regime $((r_+)_T \approx \gamma r_+)$, no analytical solution of equations (1.1) to (1.4) exists in closed form (except in the linear regime [10], where depletion of the pump beam is negligible and the pump amplitude is assumed to be independent of X). However, for a finite damping rate ($\gamma>0$), the physical mechanism responsible for soliton propagation in the hypertransient regime is expected to be valid also for transient SRS (any phase change for s_+ will lead to gain-loss reversal and solitonlike excitations).

If the coherence decay time $T_c = 1/\gamma$ is long compared to the pulse width, approximate solutions of equations (1.1) to (1.4) can be obtained through perturbation theoretic methods [11]. Under experimental conditions [1], the optical fields are given as functions of time and one wants to find their evolution in space. Kaup's general perturbation theory [11] cannot therefore be directly applied, since the soliton is defined in terms of the optical fields, while the perturbation occurs in the equations for the medium variables. A generalization to this case seems possible, and has been given for the limiting case a) above by Kaup [12]. We shall here use a more simple and direct approach which uses the following energy conservation law derived from equations (1.2) and (1.4):

$$(s_3)_X = -(r_3)_T \tag{3}$$

The basic assumption of our method is that for small γ, the analytical solutions of equations (1.1) to (1.4) have the form:

$$r_+ = r_+^0(\eta_1(X)) + r_+^1 \tag{4.1}$$

$$r_3 = r_3^0(\eta_1(X)) + r_3^1 \tag{4.2}$$

$$s_+ = s_+^0(\eta_1(X)) + s_+^1 \tag{4.3}$$

$$s_3 = s_3^0(\eta_1(X)) + s_3^1 \tag{4.4}$$

The first (zeroth order) terms are the unperturbed solutions (2.1) to (2.4) in which η_1 is assumed to be a function of X, and the second terms in the above equations are first order corrections to the one soliton solutions, which describe changes in shape.

By integrating the conservation law (3) between $T=-\infty$ and $T=+\infty$, we obtain:

$$(d/dX)\int_{-\infty}^{+\infty} s_3\, dT = -\, r_3(+\infty) - 1 \tag{5}$$

The time integral of s_3 represents the temporal width w of the optical pulse:

$$w = \int_{-\infty}^{+\infty} s_3\, dT$$

In order to calculate the asymptotic value of $r_3(+\infty)$, we use (1.1) and (1.2) to obtain:

$$(r_+^2)_T + (r_3^2)_T = -\, 2\gamma r_+^2$$

Integrating both sides from $T=-\infty$ to $T=+\infty$, and remembering that the off-diagonal matrix element should vanish at $T=+\infty$ or $-\infty$, we get the following equation:

$$r_3(+\infty)^2 - 1 = -2\gamma \int_{-\infty}^{+\infty} r_+^2 \, dT \qquad (6)$$

since $r_3(-\infty)=-1$. Equation (6) shows that the asymptotic value of r_3 at $T=+\infty$ is not equal to -1. Due to coherence decay, the perfect balance between medium excitation and deexcitation is upset: at the trailing edge of the pulse, the medium polarization is smaller than at the leading edge. As a result, a fraction of atoms gets trapped in the upper level of the Raman transition.

To first order in γ, the population difference $r_3(+\infty)$ is given by:

$$r_3(+\infty) \approx -1 + \gamma \int_{-\infty}^{+\infty} r_+^2 \, dT \qquad (7)$$

Now combining equations (5) and (7), we obtain our final result for the spatial dependence of the soliton width:

$$dw/dX = -\gamma \int_{-\infty}^{+\infty} r_+^2 \, dT \qquad (8)$$

We require that the pulse width w is entirely determined by the one soliton part s_3 of the solution:

$$w = \int_{-\infty}^{+\infty} s_3^{o} \, dT$$

To first order in γ, the right hand side of equation (8) is determined by the one soliton contribution only; hence:

$$dw/dX \approx -\gamma \int_{-\infty}^{+\infty} r_+^{o\,2} \, dT \qquad (9)$$

in which η_1 is now a function of X. The two integrals are easily calculated by using the solutions (2.1) and (2.4) and the following results are obtained:

$$dw/dX = (d/dX) \int_{-\infty}^{+\infty} (s_3^{o}+1) \, dT = d/dX \, [2\mathrm{Ln}((2\eta_1+1)/(2\eta_1-1))] \qquad (10)$$

$$\int_{-\infty}^{+\infty} r_+^{o\,2} \, dT = (1/2\eta_1^{\,3})[1+((\eta_1^{\,2}-1/4)/\eta_1)\mathrm{Ln}((2\eta_1+1)/(2\eta_1-1))] \qquad (11)$$

The temporal width of the soliton is related to the imaginary part η_1 of the eigenvalue by: $w=2\mathrm{Ln}[(2\eta_1+1)/(2\eta_1-1)]$. Equation (9) together with (10) and (11), lead to the following differential equation for the width w:

$$dw/dX = 4\gamma(\exp(w)+w\exp(w/2)-1)(2\exp(w/2)-\exp(w)-1)/(\exp(w)+2\exp(w/2)+1)^2 \qquad (12)$$

This equation has two interesting limiting cases:

* when η_1 is large (negligible medium saturation), the soliton width is inversely proportional to the square root of X:

$$w(X) \approx 2/(X + 4/w(0)^2)^{\frac{1}{2}} \qquad (\eta_1 \to \infty) \qquad (13)$$

in accordance with a previous perturbation analysis of the unsaturated case [9];

* when the medium has a large medium saturation ($\eta_1 \rightarrow 1/2$), the soliton width decreases proportionally to X:

$$w(X) \approx -4\gamma X + w(0) \qquad (\eta_1 \rightarrow 1/2) \qquad (14)$$

In order to check the accuracy of our perturbation method, we have solved equations (1.1) to (1.4) numerically; the Raman medium was assumed to be located in the half space X>0 and not excited initially; for optical pulses of finite duration, the boundary conditions were:

for T>0: $r_+(X,T=0) = 0$

$r_3(X,T=0) = 0$

$s_+(X=0,T) = s_+(X=0,T)$

$s_3(X=0,T) = s_3(X=0,T)$

for T<0: $s_+(X=0,T) = 0$

$s_3(X=0,T) = 0$

The initial fields were taken to be equal to their one soliton form at the entrance of the Raman cell (X=0) and their evolution was then calculated, as the pulses propagated into the cell (X>0), by solving (1.1) to (1.4) numerically, for different initial values of η_1 and for different small values of the homogeneous broadening parameter γ. The soliton width given by the numerical solutions for s_3 for different values of the gain length X was then compared to the analytical solution for the temporal width given by equation (12).

In fig. 1 and 2, we show how the analytical and numerical results compare for η_1 =2, γ=0.1 and η_1 =0.55, γ=0.02 respectively. The width was chosen to put the soliton well within the area of integration, yielding errors from the latter procedure exponentially small. The solid curve and the point curve are the numerical and analytical widths respectively, as a function of X, for the temporal Raman solitons. At higher initial medium saturation, the tendency towards a linear dependence of w on the propagation distance becomes [illegible]. In both cases, numerical and analytical results closely agree, showing the validity of our approximation.

We plan, in another paper, to extend our analysis to include general complex eigenvalues and effects of Stark shift. Soliton decay is expected to occur in these cases [illegible]; similarly the question of soliton generation from physically reasonable initial conditions needs to be addressed.

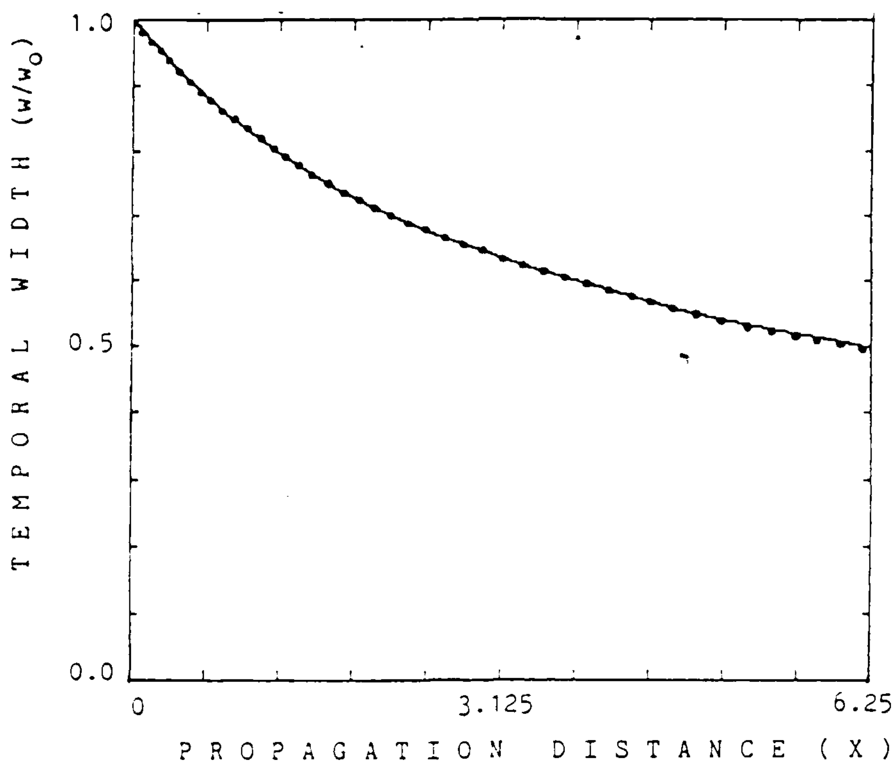

Fig 1 Temporal width of soliton (divided by initial width) as a function of propagation distance X, for $\eta_1=2$ and $\gamma=0.1$. Numerical integration (solid) and analytical results (dotted).

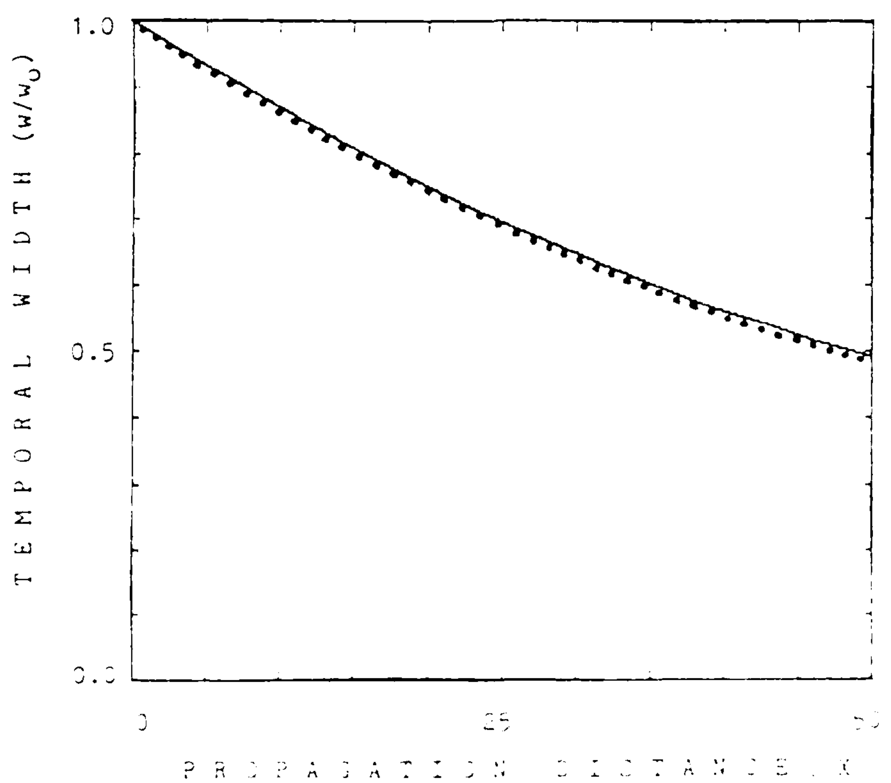

Fig.2 Temporal width of soliton (divided by initial width) as a function of propagation distance X, for η_1=[illegible] and $\gamma=0.02$. Numerical integration (solid) and analytical results (dotted).

References

1) K. J. Druhl, J. L. Carlsten, R. G. Wenzel, J. Stat. Phys. 39, 615 (1985)
2) K. J. Druhl, R. G. Wenzel, J. L. Carlsten, Phys. Rev. Lett. 51, 1171 (1983)
3) R. G. Wenzel, J. L. Carlsten, K. J. Druhl, J. Stat. Phys. 39, 621 (1985)
4) K. J. Druhl, G. Alsing, Physica D 20, 429 (1986)
5) D. J. Kaup, A. C. Newell, Proc. R. Soc. Lond. A 361, 413 (1978)
6) D. J. Kaup, Physica D 6, 143 (1983)
7) H. Steudel, Physica D 6, 155 (1983)
8) M. J. Ablowitz, D. J. Kaup, A. C. Newell, H. Segur, Stud. Appl. Math. 53, 249 (1974)
9) F. Y. F. Chu, A. C. Scott, Phys. Rev. A 12, 5, 2060 (1975)
10) R. L. Carman, F. Shimizu, C. S. Wang, N. Bloembergen, Phys. Rev. A 2, 1, 60 (1970)
11) D. J. Kaup, SIAM J. Appl. Math 31, 121 (1976)
12) D. J. Kaup, Physica D 19, 125 (1986)
13) M. Yousaf, S. Shakir, K. J. Druhl, "Propagation of complex solitons in an homogeneously broadened Raman medium: A perturbation analysis", to be published.

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